

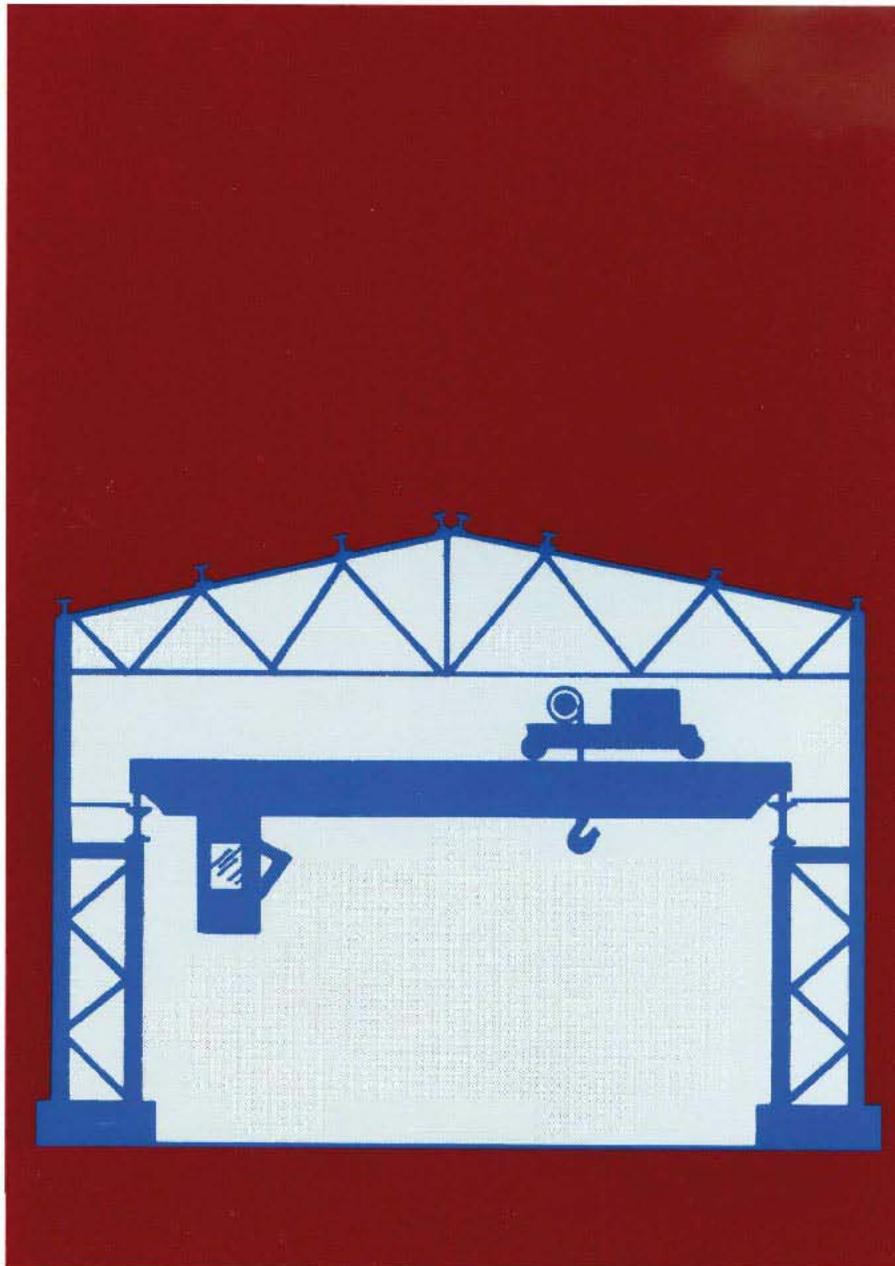


Steel Design Guide Series

7

Industrial Buildings

Roofs to Column Anchorage





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Roofs to Column Anchorage

Roofs to Column Anchorage
James M. Fisher
Computerized Structural Design, Inc.
Milwaukee, WI

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by

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Note: This Design Guide is generally based on AISC ASD provisions. Any LRFD provisions have been specifically referenced as LRFD.

PREFACE

This booklet was prepared under the direction of the Committee on Research of the American Institute of Steel Construction, Inc. as part of a series of publications on special topics related to fabricated structural steel. Its purpose is to serve as a supplemental reference to the AISC Manual of Steel Construction to assist practicing engineers engaged in building design.

The design guidelines suggested by the author that are outside the scope of the AISC Specifications or Code do not represent an official position of the Institute and are not intended to exclude other design methods and procedures. It is recognized that the design of structures is within the scope of expertise of a competent licensed structural engineer, architect or other licensed professional for the application of principles to a particular structure.

The sponsorship of this publication by the American Iron and Steel Institute is gratefully acknowledged.

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1. INTRODUCTION

Part 1

INDUSTRIAL BUILDINGS - GENERAL

Although the basic structural and architectural components of industrial buildings are relatively simple, combining all of the elements into a functional economical building can be a complex task. General guidelines and criteria to accomplish this task can be stated. The purpose of this guide is to provide the industrial building designer with guidelines and design criteria for the design of buildings without cranes, or for buildings with light-to-medium duty cycle cranes. Part 1 deals with general topics on industrial buildings. Part 2 deals with structures containing cranes.

Most industrial buildings primarily serve as an enclosure for production and/or storage. The design of industrial buildings may seem logically the province of the structural engineer. It is essential to realize however, that most industrial buildings involve much more than structural design. The designer may assume an expanded role and may be responsible for site planning, establishing grades, handling surface drainage, parking, on-site traffic, building aesthetics, and, perhaps, landscaping. Access to rail and the establishment of proper floor elevations (depending on whether direct fork truck entry to rail cars is required) are important considerations. Proper clearances to sidings and special attention to curved siding and truck grade limitations are also essential.

2. LOADING CONDITIONS AND LOADING COMBINATIONS

Loading conditions and load combinations for industrial buildings without cranes are well established by building codes.

Loading conditions are categorized as follows:

1. **Dead load:** This load represents the weight of the structure and its components, and is usually expressed in pounds per square foot. In an industrial building, the building use and industrial process usually involve permanent equipment which is supported by the structure. This equipment can sometimes be represented by a uniform load (known as a collateral load), but the points of attachment are usually subjected to concentrated loads which often require a separate analysis to account for the localized effects.

2. **Live load:** This load represents the force imposed on the structure by the occupancy and use of the building. Building codes give minimum design live loads in pounds per square foot, which vary with the classification of occupancy and use. While live loads are expressed as uniform, as a practical matter any occupancy loading is inevitably nonuniform. The degree of nonuniformity which is acceptable is a matter of engineering judgment. Some building codes deal with nonuniformity of loading by specifying concentrated loads in addition to uniform loading for some occupancies. In an industrial building, often the use of the building may require a live load in excess of the code stated minimum. Often this value is specified by the owner or calculated by the engineer. Also, the loading may be in the form of significant concentrated loads as in the case of storage racks or machinery.
3. **Snow loads:** Most codes differentiate between roof live and snow loads. Snow loads are a function of local climate, roof slope, roof type, terrain, building internal temperature, and building geometry. These factors may be treated differently by various codes.
4. **Rain loads:** These loads are now recognized as a separate loading condition. In the past, rain was accounted for in live load. However, some codes have a more refined standard. Rain loading can be a function of storm intensity, roof slope, and roof drainage. There is also the potential for rain on snow in certain regions.
5. **Wind loads:** These are well codified, and are a function of local climate conditions, building height, building geometry and exposure as determined by the surrounding environment and terrain. Typically based on a 50 year recurrence interval - maximum 5 second gust. Building codes account for increases in local pressure at edges and corners, and often have stricter standards for individual components than for the gross building. Wind can apply both inward and outward forces to various surfaces on the building exterior and can be affected by size of wall openings. Where wind forces produce overturning or net upward forces, there must be an adequate counterbalancing structural dead weight or the structure must be anchored to an adequate foundation.

6. Earthquake loads: Seismic loads are established by building codes and are based on:
 - a. the degree of seismic risk,
 - b. the degree of potential damage,
 - c. the possibility of total collapse, and
 - d. the feasibility of meeting a given level of protection.

Earthquake loads in building codes are usually equivalent static loads. Seismic loads are generally a function of:

- a. the geographical and geological location of the building,
- b. the use of the building,
- c. the nature of the building structural system,
- d. the dynamic properties of the building,
- e. the dynamic properties of the site, and
- f. the weight of the building and the distribution of the weight.

Load combinations are formed by adding the effects of loads from each of the load sources cited above. Codes or industry standards often give specific load combinations which must be satisfied. It is not always necessary to consider all loads at full intensity. Also, certain loads are not required to be combined at all. For example, wind need not be combined with seismic. In some cases only a portion of a load must be combined with other loads. When a combination does not include loads at full intensity it represents a judgment as to the probability of simultaneous occurrence with regard to time and intensity.

3. OWNER ESTABLISHED CRITERIA

Every industrial building is unique. Each is planned and constructed to requirements relating to building usage, the process involved, specific owner requirements and preferences, site constraints, cost, and building regulations. The process of design must balance all of these factors. The owner must play an active role in passing on to the designer all requirements specific to the building such as:

1. area, bay size, plan layout, aisle location, future expansion provisions,
2. clear heights,
3. relations between functional areas, production flow, acoustical considerations,

4. exterior appearance,
5. materials and finishes, etc.,
6. machinery, equipment and storage method, and
7. loads.

There are instances where loads in excess of code minimums are required. Such cases call for owner involvement. The establishment of loading conditions provides for a structure of adequate strength. A related set of criteria are needed to establish the serviceability behavior of the structure. Serviceability design considers such topics as deflection, drift, vibration and the relation of the primary and secondary structural systems and elements to the performance of nonstructural components such as roofing, cladding, equipment, etc. Serviceability issues are not strength issues but maintenance and human response considerations. Serviceability criteria are discussed in detail in Serviceability Design Considerations for Low-Rise Buildings which is part of the AISC Steel Design Guide Series⁽¹⁷⁾ Criteria taken from the Design Guide are presented in this text as appropriate.

As can be seen from this discussion, the design of an industrial building requires active owner involvement. This is also illustrated by the following topics, of slab-on-grade design, jib cranes, interior vehicular traffic, and future expansion.

3.1 Slab-on-Grade Design

One important aspect to be determined is the specific loading to which the floor slab will be subjected. Forklift trucks, rack storage systems, or wood dunnage supporting heavy manufactured items cause concentrated loads in industrial structures. The important point here is that these loadings are nonuniform. The slab-on-grade is thus often designed as a plate on an elastic foundation subject to concentrated loads.

It is common for owners to specify that slabs-on-grade be designed for a specific uniform loading (e.g., 500 psf). If a slab-on-grade is subjected to a uniform load, it will develop no bending moments. Minimum thickness and no reinforcement would be required. The frequency with which the authors have encountered the requirement of design for a uniform load and the general lack of appreciation of the inadequacy of such a criteria by many owners and plant engineers has prompted the inclusion of this topic in this guide. Real loads are not uniform, and an analysis using an assumed nonuniform load or the specific concentrated loading for the slab is required. An excellent reference for the design of slabs-on-grade is Designing Floor Slabs on Grade by Ringo and Anderson⁽³³⁾.

3.2 Jib Cranes

Another loading condition that should be considered is the installation of jib cranes. Often the owner has plans to install such cranes at some future date. But since they are a purchased item - often installed by plant engineering personnel or the crane manufacturer - the owner may inadvertently neglect them during the design phase.

Jib cranes which are simply added to a structure can create a myriad of problems, including column distortion and misalignment, column bending failures, crane runway and crane rail misalignment, and excessive column base shear. It is essential to know the location and size of jib cranes in advance, so that columns can be properly designed and proper bracing can be installed if needed. Columns supporting jib cranes should be designed to limit the deflection at the end of the jib boom to boom length divided by 225.

3.3 Interior Vehicular Traffic

The designer must establish the exact usage to which the structure will be subjected. Interior vehicular traffic is a major source of problems in structures. Forklift trucks can accidentally buckle the flanges of a column, shear off anchor bolts in column bases, and damage walls.

Proper consideration and handling of the forklift truck problem may include some or all of the following:

1. Use of masonry or concrete exterior walls in lieu of metal panels. (Often the lowest section of walls is masonry or concrete, and metal panels are used above.)
2. Installation of fender posts (bullards) for columns and walls may be required where speed and size of fork trucks are such that a column or load bearing wall could be severely damaged or collapsed upon impact.
3. Use of metal guard rails or steel plate adjacent to wall elements may be in order.
4. Curbs.

Lines defining traffic lanes painted on factory floors have never been successful in preventing structural damage from interior vehicular operations. The only realistic approach for solving this problem is to anticipate potential impact and damage and to install barriers and/or materials that can withstand such abuse.

3.4 Future Expansion

Except where no additional land is available, every industrial structure is a candidate for future expansion.

Lack of planning for such expansion can result in considerable expense.

When consideration is given to future expansion, there are a number of practical considerations that require evaluation.

1. The directions of principal and secondary framing members require study. In some cases it may prove economical to have a principal frame line along a building edge where expansion is anticipated and to design edge beams, columns and foundations for the future loads. If the structure is large and any future expansion would require creation of an expansion joint at a juncture of existing and future construction, it may be prudent to have that edge of the building consist of nonload-bearing elements. Obviously, foundation design must also include provision for expansion.
2. Roof Drainage: An addition which is constructed with low points at the junction of the roofs can present serious problems in terms of water, ice and snow piling effects.
3. Lateral stability to resist wind and seismic loadings is often provided by X-bracing in walls or by shear walls. Future expansion may require removal of such bracing. Formal notification should be made to the owner of the critical nature of wall bracing and its location to prevent accidental removal. In this context, bracing can interfere with many plant production activities and the importance of such bracing cannot be overemphasized to the owner and plant engineering personnel. Obviously, the location of bracing to provide the capability for future expansion without its removal should be the goal of the designer.

3.5 Dust Control/Ease of Maintenance

In certain buildings (e.g. food processing plants) dust control is essential. Ideally there should be no horizontal surfaces on which dust can accumulate. Tube sections as purlins reduce the number of horizontal surfaces as compared to C's, Z's, or joists. If horizontal surfaces can be tolerated in conjunction with a regular cleaning program, C's or Z's may be preferable to joists. The same thinking should be applied to the selection of main framing members (i.e. tubes or box sections may be preferable to beams or trusses).

4. ROOF SYSTEMS

The roof system is often the most expensive part of an industrial building (even though walls are more costly

per square foot). Designing for a 20 psf mechanical surcharge load when only 10 psf is required adds cost over a large area.

Often the premise which guides the design is that the owner will always be hanging new piping or installing additional equipment, and a prudent designer will allow for this in the system. If this practice is followed, the owner should be consulted, and the decision to provide excess capacity should be that of the owner. The design live loads and collateral (equipment) loads should be noted on the structural plans.

4.1 Steel Deck for Built-up or Membrane Roofs

Decks are commonly 1-1/2 in. deep, but deeper units are also available. The Steel Deck Institute⁽⁹⁾ has identified three standard profiles for 1-1/2 in. steel deck, (narrow rib, intermediate rib and wide rib) and has published load tables for each profile for thicknesses varying from .0299 to .0478 inches. These three profiles, (shown in Table 4.1) NR, IR, and WR, correspond to the manufacturers' designations A, F and B respectively. The Steel Deck Institute identifies the standard profile for 3 in. deck as 3DR. A comparison of weights for each profile in various gages shows that strength to weight ratio is most favorable for wide rib and least favorable for narrow rib deck. In general, the deck selection which results in the least weight per square foot may be the most economical. However consideration must also be given to the flute width because the insulation must span the flutes. In the northern areas of the US, high roof loads and thick insulation generally make the wide rib (B) profile predominant. In the South, low roof loads and thinner insulation make the intermediate profile common. Where very thin insulation is used narrow rib deck may be required, although this is not a common profile. In general the lightest weight deck consistent with insulation thickness and span should be used.

In addition to the load, span, and thickness relations established by the load tables, there are other considerations in the selection of a profile and gage for a given load and span. First, the Steel Deck Institute limits deflection due to a 200 lb concentrated load at midspan to span divided by 240. Secondly, the Steel Deck Institute has published a table of maximum recommended spans for construction and maintenance loads (Table 4.1), and, finally Factory Mutual lists maximum spans for various profiles and gages in its Approval Guide⁽³⁾ (Table 4.2).

Factory Mutual in its Loss Prevention Guide (LPG)I-28 Insulated Steel Deck⁽²³⁾ provides a standard for attachment of insulation to steel deck. LPG 1-29 Loose Laid Ballasted Roof Coverings⁽²³⁾ gives a standard for the required weight and distribution of ballast for roofs that are not adhered.

LPG 1-28 requires a side lap fastener between supports. This fastener prevents adjacent panels from deflecting differentially when a load exists at the edge of one panel but not on the edge of the adjacent panel. Factory Mutual permits an over span from its published tables of 6 inches (previously an overspan of 10 percent had been allowed) when "necessary to accommodate column spacing in some bays of the building. It should not be considered an original design parameter." The Steel Deck Institute recommends that the side laps in cantilevers be fastened at twelve inches on center.

Steel decks can be attached to supports by welds or fasteners, which can be power or pneumatically installed or self-drilling, self-tapping. The Steel Deck Institute in its Specifications and Commentary for Steel Roof Deck⁽³⁸⁾ requires a maximum attachment spacing of 18 in. along supports. Factory Mutual requires the use of 12 in. spacing as a maximum; this is more common. The attachment of roof deck must be sufficient to provide bracing to the structural roof members, to anchor the roof to prevent uplift, and, in many cases, to serve as a diaphragm to carry lateral loads to the bracing. While the standard attachment spacing may be acceptable in many cases decks designed as diaphragms may require additional connections. Diaphragm capacities can be determined through use of Ref. 11.

Manufacturers of metal deck are constantly researching ways to improve section properties with maximum economy. Considerable differences in cost may exist between prices from two suppliers of "identical" deck shapes; therefore the designer is urged to research the cost of the deck system carefully. A few cents per square foot savings on a large roof area can mean a significant savings to the owner.

Several manufacturers can provide steel roof deck and wall panels with special acoustical surface treatments for specific building use. Properties of such products can be obtained from the manufacturers. Special treatment for acoustical reasons must be specified by the owner.

4.2 Metal Roofs

Standing seam roof systems were first introduced in the late 1960's, and today many manufacturers produce standing seam panels. A difference between the standing seam roof and lap seam roof (through fastener roof) is in the manner in which two panels are joined to each other. The seam between two panels is made in the field with a tool that makes a cold formed weather-tight joint. (Note: some panels can be seamed without special tools.) The joint is made at the top of the panel. The standing seam roof is also unique in the manner in which it is attached to the purlins. The attachment is made with a clip concealed inside the seam. This clip secures the panel to the purlin

| Recommended Maximum Spans for Construction and Maintenance Loads Standard 1-1/2 Inch and 3 Inch Roof Deck | | | | |
|---|-------|----------------|--------------|--|
| | Type | Span Condition | Span Ft.-In. | Maximum Recommended Spans Roof Deck Cantilever |
| Narrow Rib Deck (Old Type A) | NR22 | 1 | 3'-10" | 1'-0" |
| | NR22 | 2 or more | 4'-9" | |
| | NR20 | 1 | 4'-10" | 1'-2" |
| | NR20 | 2 or more | 5'-11" | |
| | NR18 | 1 | 5'-11" | 1'-7" |
| | NR18 | 2 or more | 6'-11" | |
| Intermediate Rib Deck (Old Type F) | IR22 | 1 | 4'-6" | 1'-2" |
| | IR22 | 2 or more | 5'-6" | |
| | IR20 | 1 | 5'-3" | 1'-5" |
| | IR20 | 2 or more | 6'-3" | |
| | IR18 | 1 | 6'-2" | 1'-10" |
| | IR18 | 2 or more | 7'-4" | |
| Wide Rib (Old Type B) | WR22 | 1 | 5'-6" | 1'-11" |
| | WR22 | 2 or more | 6'-6" | |
| | WR20 | 1 | 6'-3" | 2'-4" |
| | WR20 | 2 or more | 7'-5" | |
| | WR18 | 1 | 7'-6" | 2'-10" |
| | WR18 | 2 or more | 8'-10" | |
| Deep Rib Deck | 3DR22 | 1 | 11'-0" | 3'-6" |
| | 3DR22 | 2 or more | 13'-0" | |
| | 3DR20 | 1 | 12'-6" | 4'-0" |
| | 3DR20 | 2 or more | 14'-8" | |
| | 3DR18 | 1 | 15'-0" | 4'-10" |
| | 3DR18 | 2 or more | 17'-8" | |

NOTE: SEE SDI LOAD TABLES FOR ACTUAL DECK CAPACITIES.

Table 4.1 Steel Deck Institute Recommended Spans⁽³⁸⁾

| Types 1.5A, 1.5F, 1.5B and 1.5BI Deck. Nominal 1-1/2 in. (38 mm) depth. No stiffening grooves. | | | | |
|--|-----------------|----------------|----------------|--|
| | 22g. | 20g. | 18g. | |
| Type 1.5A Narrow Rib | 4'10" (1.5m) | 5'3" (1.6m) | 6'0" (1.9m) | |
| Type 1.5F Intermediate Rib | 4'11" (1.5m) | 5'5" (1.7m) | 6'3" (2.0m) | |
| Type 1.5B, BI Wide Rib | 6'0" (1.8m) | 6'6" (2.0m) | 7'5" (2.3m) | |

Table 4.2 Factory Mutual Data⁽³⁾

and may allow the panel to move when experiencing thermal expansion or contraction.

A continuous single skin membrane results after the seam is made since through-the-roof fasteners have been eliminated. The elevated seam and single skin member provides a watertight system. The ability of the roof to experience unrestrained thermal movement eliminates damage to insulation and structure (caused by temperature effects which built-up and through fastened roofs commonly experience). Thermal spacer blocks are often placed between the panels and purlins in order to insure a consistent thermal barrier. Due to the superiority of the standing seam roof, most manufacturers are willing to offer considerably longer guarantees than those offered on lap seam roofs.

Because of the ability of standing seam roofs to move on sliding clips, they possess only minimal diaphragm strength and stiffness. The designer should as-

sume that the standing seam roof has no diaphragm capability, and in the case of steel joists specify that sufficient bridging be provided to laterally brace the joists under design loads.

4.3 Insulation and Roofing

Due to concern about energy, the use of additional and/or improved roof insulation has become common. Coordination with the mechanical requirements of the building are necessary. Generally the use of additional insulation is warranted, but there are at least two practical problems that occur as a result. Less heat loss through the roof results in greater snow and ice build-up and larger snow loads. As a consequence of the same effect, the roofing is subjected to colder temperatures and, for some systems (built-up roofs), thermal movement, which may result in cracking of the roofing membrane.

4.4 Expansion Joints

Although industrial buildings are often constructed of flexible materials, roof and structural expansion joints are required when horizontal dimensions are large. It is not possible to state exact requirements relative to distances between expansion joints because of the many variables involved, such as ambient temperature during construction and the expected temperature range during the life of the buildings. An excellent reference on the topic of thermal expansion in buildings and location of expansion joints is the Federal Construction Council's Technical Report No. 65, Expansion Joints in Buildings.⁽¹⁴⁾

The report presents the figure shown herein as Figure 4.4.1 as a guide for spacing structural expansion joints in beam and column frame buildings based on design temperature change. The report includes data for numerous cities. The report gives modifying factors which are applied to the allowable building length as appropriate.

The report indicates that the curve is directly applicable to buildings of beam-and-column construction, hinged at the base, and with heated interiors. When other conditions prevail, the following rules are applicable:

1. If the building will be heated only and will have hinged-column bases, use the allowable length as specified.
2. If the building will be air conditioned as well as heated, increase the allowable length 15 percent (if the environmental control system will run continuously).
3. If the building will be unheated, decrease the allowable length 33 percent.

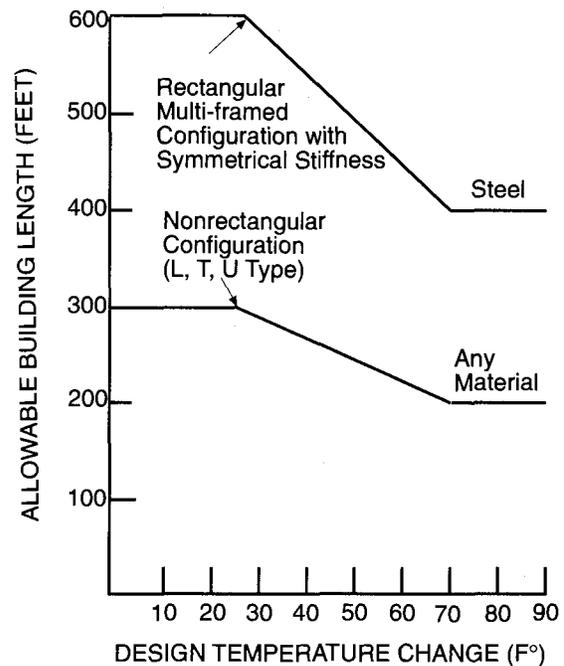


Fig. 4.4.1 Expansion Joint Spacing Graph

[Taken from F.C.C. Tech. Report No. 65, Expansion Joints in Buildings]

4. If the building will have fixed column bases, decrease the allowable length 15 percent.
5. If the building will have substantially greater stiffness against lateral displacement in one direction decrease the allowable length 25 percent.

When more than one of these design conditions prevail in a building, the percentile factor to be applied should be the algebraic sum of the adjustment factors of all the various applicable conditions.

Regarding the type of structural expansion joint, most engineers agree that the best method is to use a line of double columns to provide a complete separation at the joints. When joints other than the double column type such as Fig. 4.4.2 are employed, low friction sliding elements are generally used. Slip connections may induce some level of inherent restraint to movement due to binding or debris build-up.

Very often buildings may be required to have fire walls in specific locations. Fire walls may be required to extend above the roof or it may be allowed to terminate at the underside of the roof. Such fire walls become locations for expansion joints. In such cases the detailing of joints can be difficult.

Figures 4.4.2 through 4.4.5 depict typical details to permit limited expansion. Additional details are given in architectural texts.

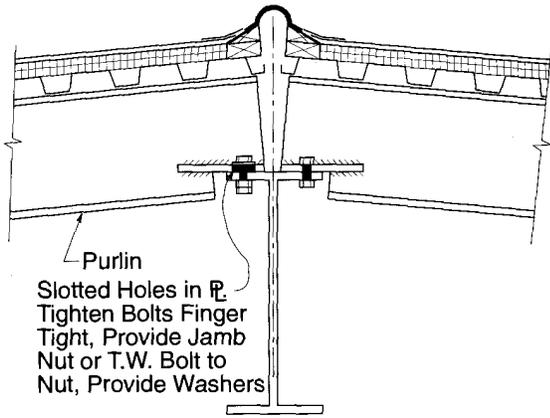


Fig. 4.4.2 Beam Expansion Joint

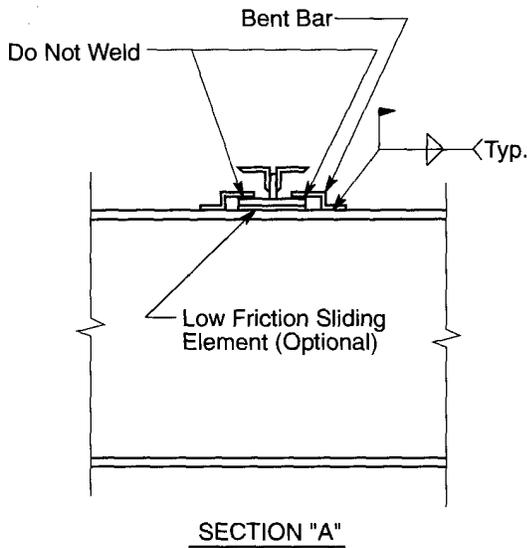
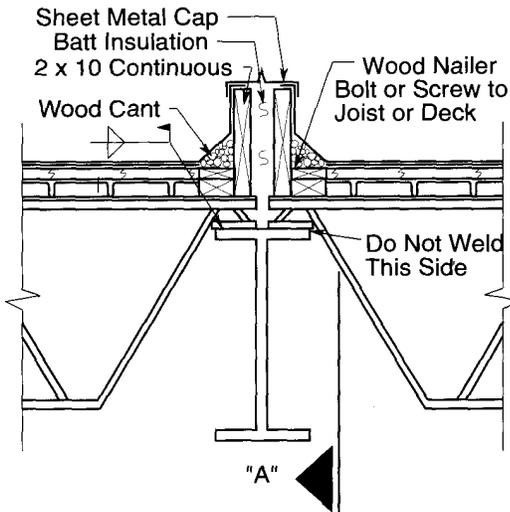


Fig. 4.4.3 Joist Expansion Joint

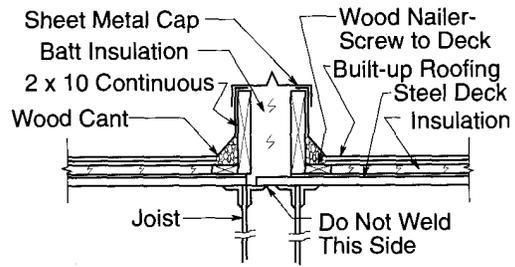


Fig. 4.4.4 Joist Expansion Joint

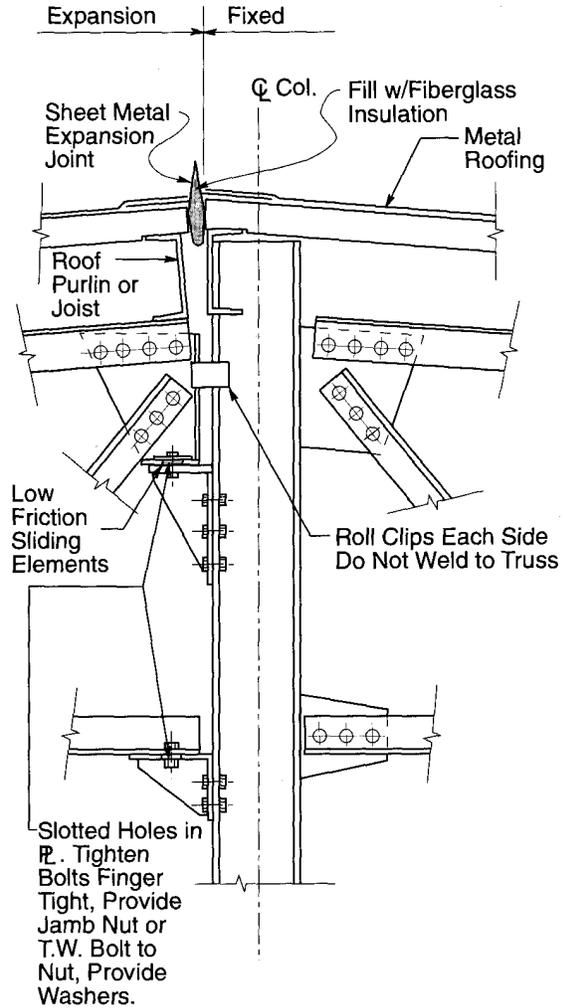


Fig. 4.4.5 Truss Expansion Joint

Expansion joints in the structure should always be carried through the roofing. Additionally, depending on membrane type, other joints called area dividers are necessary in the roof membrane. These joints are membrane relief joints only and do not penetrate the roof deck. Area divider joints are generally placed at intervals of 150-250 feet for adhered membranes, at somewhat greater intervals for ballasted membranes, and 100-200 feet in the case of steel roofs. Spacing of joints should be

verified with manufacturer's requirements. The range of movement between joints is limited by the flexibility and movement potential of the anchorage scheme and, in the case of standing seam roofs, the clip design. Manufacturers' recommendations should be consulted and followed. Area dividers can also be used to divide complex roofs into simple squares and rectangles.

4.5 Roof Pitch, Drainage and Ponding

Prior to determining a framing scheme and the direction of primary and secondary framing members, it is important to decide how roof drainage is to be accomplished. If the structure is heated, interior roof drains may be justified. For unheated spaces exterior drains and gutters may provide the solution.

For some building sites it may not be necessary to have gutters and downspouts, to control stormwater but their use is generally recommended or required by the owner. Significant operational and hazardous problems can occur where water is discharged at the eaves or scuppers in cold climates, causing icing of ground surfaces and hanging of ice from the roof edge. This is a special problem at overhead door locations and may occur with or without gutters. Protection from falling ice must be provided at all building service entries.

Performance of roofs with positive drainage is generally good. Problems (e.g., ponding, roofing deterioration, leaking) which may result from poor drainage lead to the recommendation that a roof slope of at least 1/4 in. per ft. be provided for all building roof systems. Ponding, which is often not understood or is overlooked, is a phenomenon that may lead to severe distress or partial or general collapse.

Ponding as it applies to roof design has two meanings. To the roofing industry, ponding describes the condition in which water accumulated in low spots has not dissipated within 24 hours of the last rain storm. Ponding of this nature is addressed in roof design by positive roof drainage and control of the deflections of roof framing members. Ponding an issue in structural engineering is a load/deflection situation in which there is incremental accumulation of rain water in the deflecting structure. The purpose of a ponding check is to insure that an equilibrium is reached between the incremental loading and the incremental deflection. This convergence must occur at a level of stress that is within the allowable value.

The AISC Specifications for both LRFD⁽²²⁾ and ASD⁽⁴²⁾ give procedures for addressing the problem of ponding where roof slopes and drains may be inadequate. The direct method is expressed in Eq. K2-1 and K2-2 of the Specifications. These relations control the stiffness of the framing members (primary and secondary) and deck. This method, however, can produce unnecessarily

conservative results. A more exact method is provided in Appendix K of the LRFD Specification and in Chap. K in the Commentary in the ASD Specification.

The key to the use of the allowable stress method is the calculation of stress in the framing members due to loads present at the initiation of ponding. The difference between $0.8 F_y$ and the initial stress is used to establish the required stiffness of the roof framing members. The initial stress ("at the initiation of ponding") is determined from the loads present at that time. These should include all or most of the dead load and may include some portion of snow/rain/live load. Technical Digest No. 3 published by the Steel Deck Institute⁽⁴¹⁾ gives some guidance as to the amount of snow load which could be used in ponding calculations.

The amount of accumulated water used is also subject to judgment. The AISC Ponding criteria only applies to roofs which lack "... sufficient slope towards parts of free drainage or adequate individual drains to prevent the accumulation of rain water...". However the possibility of plugged drains means that the load at the initiation of ponding could include the depth of impounded water at the level of overflow into adjacent bays, or the elevation of overflow drains or, over the lip of roof edges or through scuppers. It is clear from reading the AISC Specification and Commentary that it is not necessary to include the weight of water which would accumulate after the "initiation of ponding". Where snow load is used by the code, the designer may add 5 psf to the roof load to account for the effect of rain on snow. Also, consideration must be given to areas of drifted snow.

It is clear that judgment must be used in the determination of loading "at the initiation of ponding". It is equally clear that one hundred percent of the roof design load would rarely be appropriate for the loading "at the initiation of ponding".

A continuously framed or cantilever system may be more critical than a simple span system. With continuous framing, rotations at points of support, due to non-uniformly distributed roof loads, will initiate upward and downward deflections in alternate spans. The water in the uplifted bays drains into the adjacent downward deflected bays, compounding the effect and causing the downward deflected bays to approach the deflected shape of simple spans. For these systems one approach to ponding analysis could be based on simple beam stiffness, although a more refined analysis could be used.

The designer should also consult with the plumbing designer to establish whether or not a controlled flow (water retention) drain scheme is being used. Such an approach allows the selection of smaller pipes because the water is impounded on the roof and slowly drained away. This intentional impoundment does not meet the AISC

criterion of "... drains to prevent the accumulation of rainwater..." and requires a ponding analysis.

Besides rainwater accumulation, the designer should give consideration to excessive build-up of material on roof surfaces (fly ash, and other air borne material) from industrial operations. Enclosed valleys, parallel high and low aisle roofs and normal wind flows can cause unexpected build-ups and possibly roof overload.

4.6 Joists and Purlins

A decision must be made whether to span the long direction of bays with the main beams trusses or joist girders which support short span joists or purlins, or to span the short direction of bays with main framing members which support longer span joists or purlins. Experience in this regard is that spanning the shorter bay dimension with primary members will provide the most economical system. However, this decision may not be based solely on economics but rather on such factors as ease of erection, future expansion, direction of crane runs, location of overhead doors, etc.

On the use of steel joists or purlins, experience again shows that each case must be studied. Standard steel joist specifications (See Ref. 40) are based upon distributed loads only. Modifications for concentrated loads should be done in accordance with the SJI Code Of Standard Practice. Significant concentrated loads should be supported by hot rolled framing members. However in the absence of large concentrated loads, joist framing can generally be more economical than hot rolled framing.

Cold-formed C and Z purlin shapes provide another alternative to rolled W sections. The provisions contained in the American Iron and Steel Institute's (AISI) Specification for the Design of Cold-Formed Steel Structural Members⁽⁴³⁾ or the AISI Load and Resistance Factor Design Specification for Cold-Formed Steel Structural Members⁽²¹⁾ should be used for the design of cold formed purlins. Additional economy can be achieved with C and Z sections because they can be designed and constructed as continuous members. However, progressive failure should be considered if there is a possibility for a loss in continuity after installation.

Other aspects of the use of C and Z sections include:

1. Z sections ship economically due to the fact that they can be "nested".
2. Z sections can be loaded through the shear center, C sections cannot.
3. On roofs with appropriate slope a Z section will have one principal axis vertical, while a C section provides this condition only for flat roofs.

4. Many erectors indicate that lap bolted connections for C or Z sections (bolts) are more expensive than the simple welded down connections for joist ends.
5. At approximately a 30 ft. span length C and Z sections may cost about the same as a joist for the same load per foot. For shorter spans C and Z sections may normally be less expensive than joists.

5. ROOF TRUSSES

Primary roof framing for conventionally designed industrial buildings generally consists of wide flange beams, steel joist girders, or fabricated trusses. For relatively short spans 30 to 40 feet steel beams provide an economical solution, particularly if a multitude of hanging loads are present. For spans greater than 40 but less than 80 feet steel joist girders are often used to support roof loads. Fabricated steel roof trusses are often used for spans greater than 80 feet. In recent years little has been written about the design of steel roof trusses. Most textbooks addressing the design of trusses were written when riveted connections were used. Today welded trusses and field bolted trusses are used exclusively. Presented in the following paragraphs are concepts and principles that apply to the design of roof trusses.

5.1 General Design and Economic Considerations

No absolute statements can be made about what truss configuration will provide the most economical solution for a particular situation; however, the following statements can be made regarding truss design:

1. Span-to-depth ratios of 15 to 20 generally prove to be economical; however, shipping depth limitations should be considered so that shop fabrication can be maximized. The maximum depth for shipping is conservatively about 14 feet. Greater depths will require the web members to be field bolted which will increase erection costs.
2. The length between splice points is also limited by shipping lengths. The maximum shippable length varies according to the destination of the trusses; but lengths of 80 feet are generally shippable and 100 feet is often possible. Since maximum available mill length is approximately 70 feet, the distance between splice points is normally set at 70 feet. Greater distances between splice points will generally require truss chords to be shop spliced.
3. In general, the rule "deeper is cheaper" is true; however, the costs of additional lateral bracing for more flexible truss chords must be carefully

- examined relative to the cost of larger chords which may require less lateral bracing. The lateral bracing requirements for the top and bottom chords should be considered interactively while selecting chord sizes and types. Particular attention should be paid to loads which produce compression in the bottom chord. In this condition additional chord bracing will most likely be necessary.
4. If possible, select truss depths so that tees can be used for the chords rather than wide flange shapes. Tees can eliminate (or reduce) the need for gusset plates.

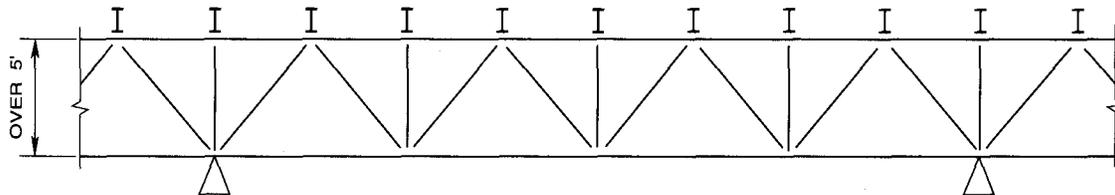


Fig. 5.1.1 Economical Truss Web Arrangement

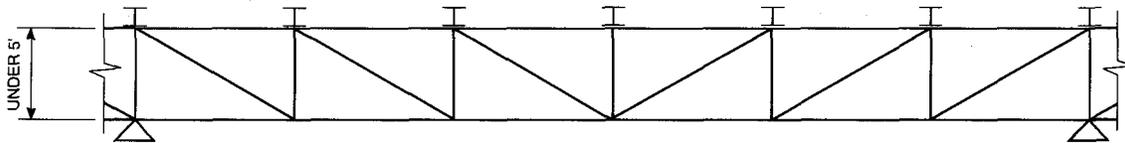


Fig. 5.1.2 Economical Truss Web Arrangement

8. Tube, wide flange or pipe sections may prove to be more effective web members at some web locations especially where subsystems are to be supported by web members.
9. Designs using the AISC LRFD Specification⁽²²⁾ will often lead to truss savings when heavy long span trusses are required. This is due to the higher DL to LL ratios for these trusses.
10. The weight of gusset plates, shim plates and bolts can be significant in large trusses. This weight must be considered in the design since it often approaches 10 to 15 percent of the truss weight.
11. If trusses are analyzed using frame analysis computer programs and rigid joints are assumed, secondary bending moments will show up in the analysis. The reader is referred to Reference 27 wherein it is suggested that so long as these secondary stresses do not exceed 4,000 psi they may be neglected. Secondary stresses should not be neglected if the beneficial effects

5. Higher strength steels ($F_y = 50$ ksi or above) usually provides the use of more efficient truss members.
6. Illustrated in Figs. 5.1.1 and 5.1.2 are web arrangements which generally provide economical web systems.
7. Utilize only a few web angle sizes, and make use of efficient long leg angles for greater resistance to buckling. Differences in angle sizes should be recognizable. For instance avoid using an angle $4 \times 3 \times 1/4$ and an angle $4 \times 3 \times 5/16$ in the same truss.

of continuity are being considered in the design process, e.g. effective length determination. The designer must be consistent. That is, if the joints are considered as pins for the determination of forces, then they should also be considered as pins in the design process. The assumption of rigid joints in some cases may provide unconservative estimates on the deflection of the truss.

12. Repetition is beneficial and economical. Use as few different truss depths as possible. It is cheaper to vary the chord size as compared to the truss depth.
13. Wide flange chords with gussets may be necessary when significant bending moments exist in the chords (i.e.: subsystems not supported at webs or large distances between webs).
14. AISC Engineering for Steel Construction⁽¹³⁾ can provide some additional guidance on truss design and detailing.
15. Design and detailing of long span joists and joist girders shall be in accordance with SJI specifications.⁽⁴⁰⁾

5.2 Connection Considerations

1. As mentioned above, tee chords are generally economical since they can eliminate gusset plates. The designer should examine the connection requirements to determine if the tee stem is in fact long enough to eliminate gusset requirements. The use of a deeper tee stem is generally more economical than adding numerous gusset plates even if this means an addition in overall weight.
2. Block shear requirements and the effective area in compression should be carefully checked in tee stems and gussets (AISC Appendix B). Shear rupture of chord members at panel points should also be investigated since this can often control wide flange chords.
3. Intermediate connectors (stitch fasteners or fillers) may be required for double web members. Examples of intermediate connector evaluation can be found in the AISC Manual under Column Design; Double Angles.
4. If wide flange chords are used with wide flange web members it is generally more economical to orient the chords with their webs horizontal. Gusset plates for the web members can then be either bolted or welded to the chord flanges. To eliminate the cost of fabricating large shim or filler plates for the diagonals, the use of comparable depth wide flange diagonals should be considered.
5. When trusses require field bolted joints the use of slip-critical bolts in conjunction with over-size holes will allow for erection alignment. Also if standard holes are used with slip-critical bolts and field "fit-up" problems occur, holes can be reamed without significantly reducing the allowable bolt shears.
6. For the end connection of trusses, top chord seat type connections should also be considered. Seat connections allow more flexibility in correcting column truss alignment during erection. Seats also provide for efficient erection and are more stable during erection than "bottom bearing" trusses. When seats are used, a simple bottom chord connection is recommended to prevent the truss from rolling during erection.
7. For symmetrical trusses use a center splice to simplify fabrication even though forces may be larger than for an offset splice.
8. End plates can provide efficient compression splices.

9. It is often less expensive to locate the work point of the end diagonal at the face of the supporting member rather than designing the connection for the eccentricity between the column Centerline and the face of the column.

5.3 Truss Bracing

Stability bracing is required at discrete locations where the designer assumes braced points or where braced points are required in the design of the members in the truss. These locations are generally at panel points of the trusses and at the ends of the web members. To function properly the braces must have sufficient strength and stiffness. Using standard bracing theory, the brace stiffness required (Factor of Safety = 2.0) is equal to $4P/L$, where P equals the force to be braced and L equals the unbraced length of the column. The required brace force equals $.004P$. As a general rule the stiffness requirement will control the design of the bracing unless the bracing stiffness is derived from axial stresses only. Braces which displace due to axial loads only are very stiff, and thus the strength requirement will control. It should be noted that the AISE Technical Report No. 13 requires a $.025P$ force requirement for bracing. More refined bracing equations are contained in Reference 20. Requirements for truss bottom chord bracing are discussed in Reference 15. These requirements do not necessarily apply to long span joists or joist girders.

Designers are often concerned about the number of "out-of-straight" trusses that should be considered for a given bracing situation. No definitive rules exist; however, the Australian Code indicates that no more than seven out of straight members need to be considered. For columns, the International Standards Code of steel structure design (ISO) has postulated the equation:

$$F_n = F_1 (.2n + .8\sqrt{n})$$

where F_n is the assumed brace force for n columns, F_1 equals $.02N$ where N is the axial force in the column. This equation thus suggests that $(.2n + .8\sqrt{n})$ trusses should be considered in the bracing design. Thus, if ten trusses were to be braced, bracing forces could be based on five trusses. Common practice is to provide horizontal bracing every five to six bays to transfer bracing forces to the main force resisting system. In this case the brace forces should be calculated based on the number of trusses between horizontal bracing.

A convenient approach to the stability bracing of truss compression chords is discussed in Reference 28. The solution presented is based upon the brace stiffness requirements controlled by an X-braced system. The paper indicates that as long as the horizontal X-bracing system is comprised of axially loaded members arranged as shown in Fig. 5.3.1, the bracing can be designed for 0.6 percent of the truss chord axial load. Since two truss

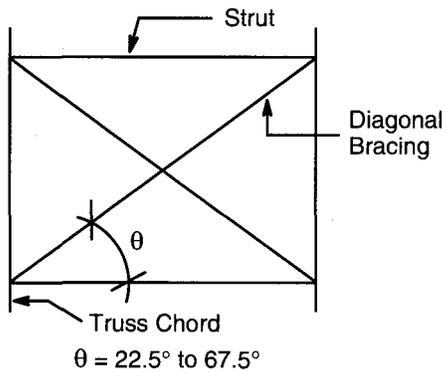


Fig. 5.3.1 Horizontal X-Bracing Arrangement

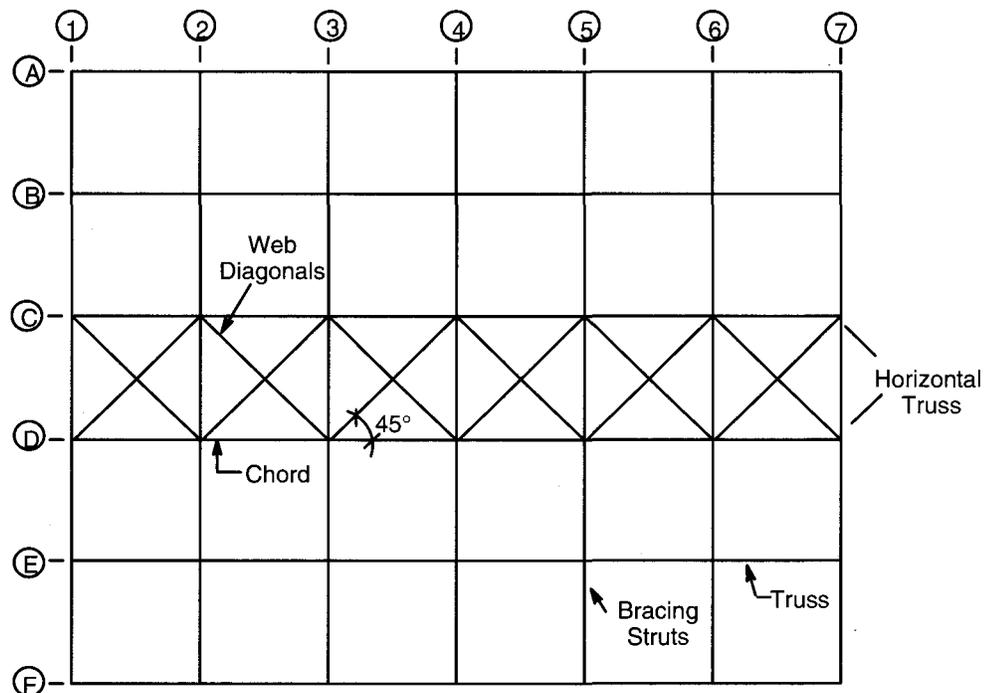
chord sections are being braced at each bracing strut location the strut connections to the trusses must be designed for 1.2 percent of the average chord axial load for the two adjacent chords. In the reference it is pointed out that the bracing forces do not accumulate along the

length of the truss; however the brace force requirements do accumulate based on the number of trusses considered braced by the bracing system.

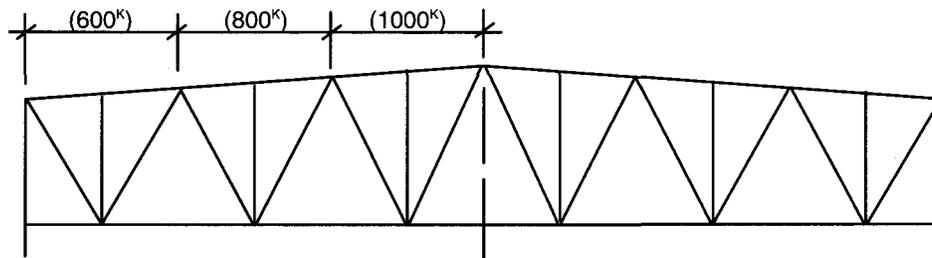
In addition to stability bracing, top and bottom chord bracing may also be required to transfer wind or seismic lateral loads to the main lateral stability system. The force requirements for the lateral loads must be added to the stability force requirements. Lateral load bracing is placed in either the plane of the top chord or the plane of the bottom chord, but generally not in both planes. Stability requirements for the unbraced plane can be transferred to the laterally braced plane by using vertical sway braces.

EXAMPLE 5.3.1: Roof Truss Stability Bracing

For the truss system shown in Fig. 5.3.2 determine the brace forces in the horizontal bracing system. Use the procedure discussed in Reference 28.



Framing Plan



Truss Elevation

Fig. 5.3.2 Horizontal Bracing System

Solution:

Since the diagonal bracing layout as shown in Fig. 5.3.2 forms an angle of 45 degrees with the trusses, the

solution used in Reference 28 is suitable. The bracing force thus equals .6% of the chord axial load. Member forces are summarized below.

| DESIGN FORCES (KIPS) | | |
|------------------------------------|-----------------------------------|-------------------------------|
| HORIZONTAL TRUSS WEB MEMBER FORCES | | |
| MEMBER | PANEL SHEAR | FORCE = (1.414) (PANEL SHEAR) |
| C1-D2 D1-C2 | .006(6x600) = 21.6 | 30.5 |
| C2-D3 D2-C3 | .006(6x800) = 28.8 | 40.7 |
| C3-D4 D3-C4 | .006(6x1000) = 36 | 50.9 |
| HORIZONTAL TRUSS CHORD FORCES | | |
| MEMBER | MEMBER FORCE | |
| C1-C2 D1-D2 | 21.6 | |
| C2-C3 D2-D3 | 21.6 + 28.8 = 50.4 | |
| C3-C4 D3-D4 | 50.4 + 36 = 86.4 | |
| STRUT FORCES | | |
| MEMBER | FORCE = (1.2%) (AVE. CHORD FORCE) | |
| A4-B4, E4-F4 | 12.0 | |
| B4-C4, D4-E4 | 24.0 | |
| C4-D4 | 36.0 | |
| A3-B3, E3-F3 | 10.8 | |
| B3-C3, D3-E3 | 21.6 | |
| C3-D3 | 32.4 | |
| A2-B2, E2-F2 | 8.4 | |
| B2-C2, D2-E2 | 16.8 | |
| C2-D2 | 25.2 | |
| A1-B1, E1-F1 | 3.6 | |
| B1-C1, D1-E1 | 7.2 | |
| C1-D1 | 10.8 | |

Note: Forces not shown are symmetrical

5.4 Erection Bracing

The engineer of record is not responsible for the design of erection bracing unless specific contract arrangements incorporate this responsibility into the work. Even though the designer of trusses is not responsible for the erection bracing, the designer should consider sequence and bracing requirements in the design of large trusses in order to provide the most cost effective system. Large trusses require significant erection bracing not only to resist wind and construction loads but also to provide stability until all of the gravity load bracing is installed. Significant cost savings can be achieved if the required erection bracing is incorporated into the permanent bracing system.

Erection is generally accomplished by first connecting two trusses together with strut braces and any additional erection braces to form a stable box system. Additional trusses are held in place by the crane or cranes until they can be "tied off with strut braces to the already erected stable system. Providing the necessary components to facilitate this type of erection sequence is essential for a cost effective project.

Additional considerations are as follows:

1. Columns are usually erected first with the lateral bracing system (see Fig. 5.4.1). If top chord seats are used, the trusses can be quickly positioned on top of the columns, braced to one another.

Bottom chord bearing trusses require that additional stability bracing be installed at ends of trusses while the cranes hold the trusses in place. This can slow down the erection sequence. These concepts are shown in Fig.5.4.1.

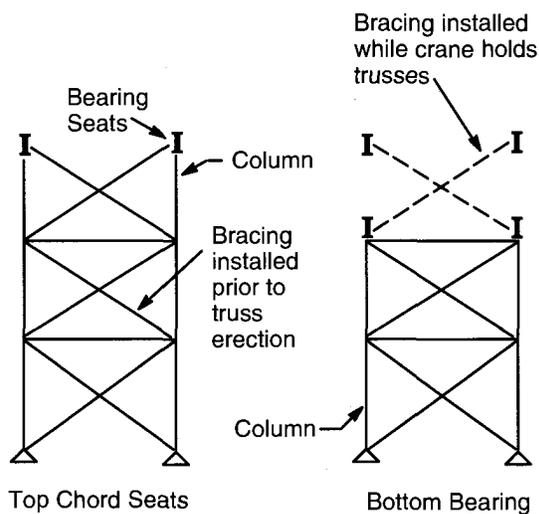


Fig. 5.4.1 Wall Bracing Erection Sequence

2. Since many industrial buildings require clear spans, systems are often designed as rigid frames. By designing rigid frames, erection is facilitated, in that, the side wall columns are stabilized in the plane of the trusses once the trusses are adequately anchored to the columns. This scheme may require larger columns than a braced frame system; however, economy can generally be recovered due to a savings in bracing and erection time.
3. Wide flange beams, tubes or pipe sections should be used to laterally brace large trusses at key locations during erection because of greater stiffness. Steel joists can be used; however, two notes of caution are advised:
 - a. Erection bracing strut forces must be provided to the joist manufacturer; and it must be made clear whether joist bridging and roof deck will not be in place when the erection forces are present. Large angle top chords in joists may be required to control the joist Slenderness ratio so that it does not buckle while serving as the erection strut,
 - b. Joists are often not fabricated to exact lengths and long slotted holes are generally provided in joist seats. Slotted holes for bolted bracing members should be avoided because of possible slippage. Special coordination with the joist manufacturer is required to eliminate the slots and to provide a suitable joist for bracing. In addition the joists must be at the job site when the erector wishes to erect the trusses.
4. Wind forces on the trusses during erection can be considerable. The AISC Code of Standard Practice states that "temporary supports will secure the steel framing, or any partly assembled steel framing, against loads comparable in intensity to those for which the structure was designed, resulting from wind, seismic forces and erection operations..." The projected area of all of the truss, and other roof framing members can be significant and in some cases the wind forces on the unsided structure are actually larger than those after the structure is enclosed.
5. A sway frame is normally required in order to plumb the trusses during erection. These sway frames should normally occur every fourth or fifth bay. An elevation view of such a truss is shown in Fig. 5.4.2. These frames can be incorporated into the bottom chord bracing system. Sway frames are also often used to transfer forces from one chord level to another as discussed earlier. In these cases the sway frames must not only be designed for stability forces, but also the required load transfer forces.

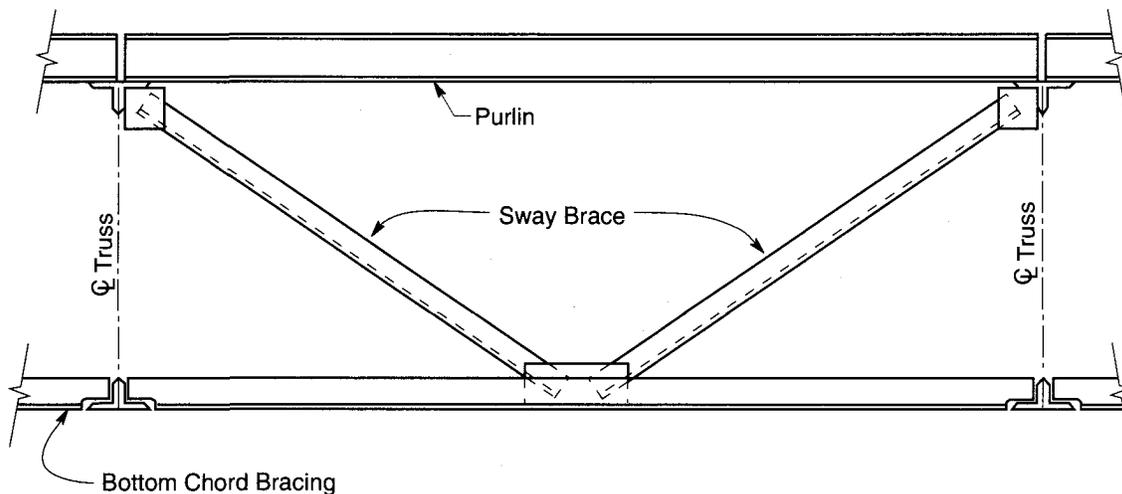


Fig. 5.4.2 Sway Frame

5.5 Other Considerations

1. Camber: Large clear span trusses are generally cambered to accommodate dead load deflections. This is accomplished by the fabricator, by either adjusting the length of the web members in the truss and keeping the top chord segments straight or by curving the top chord. Tees can generally be easily curved during assembly whereas wide flange sections may require cambering prior to assembly. If significant top chord pitch is provided and if the bottom chord is pitched, camber may not be required. The engineer of record is responsible for providing the fabricator with the anticipated dead load deflection and special cambering requirements.

The designer must carefully consider the truss deflection and camber adjacent to walls, or other portions of the structure where stiffness changes cause variations in deflections. This is particularly true at building endwalls, where differential deflections may damage continuous purlins or connections.

2. Connection details which can accommodate temperature changes are generally necessary. Long span trusses which are fabricated at one temperature and erected at a significantly different temperature can grow or shrink significantly.
3. Roof deck diaphragm strength and stiffness is commonly used for strength and stability bracing for joists. The diaphragm capabilities must be carefully evaluated if it is to be used for bracing of large clear span trusses.

6. WALL SYSTEMS

The wall system can be chosen for a variety of reasons and the cost of the wall can vary by as much as a factor of three. Wall systems include:

1. field assembled metal panels,
2. factory assembled metal panels,
3. precast concrete panels,
4. masonry walls (part or full height).

A particular wall system may be selected over others for one or more specific reasons including:

1. cost,
2. appearance,
3. ease of erection,
4. speed of erection,
5. insulating properties,
6. fire considerations,
7. acoustical considerations,
8. ease of maintenance/cleaning,
9. ease of future expansion,
10. durability of finish,
11. maintenance considerations.

Some of these factors will be discussed in the following sections on specific systems. Other factors are not discussed and require evaluation on a case by case basis.

6.1 Field Assembled Panels

Field assembled panels consist of an outer skin element, insulation, and in some cases an inner liner panel. The panels vary in material thickness and are normally galvanized, galvanized prime painted suitable for field painting, or prefinished galvanized. Corrugated aluminum liners are also used. When aluminum materials are used their compatibility with steel supports should be verified with the manufacturer since aluminum may cause corrosion of steel. When an inner liner is used, some form of hat section interior subgirts are generally provided for stiffness. The insulation is typically fiberglass or a foam. If the inner liner sheet is used as the vapor barrier all joints and edges should be sealed.

Specific advantages of field assembled wall panels include:

1. rapid erection of panels,
2. good cost competition, with a large number of manufacturers and contractors being capable of erecting panels,
3. quick and easy panel replacement in the event of panel damage,
4. openings for doors and windows that can be created quickly and easily,
5. panels that are lightweight, so that heavy equipment is not required for erection. Also large foundations and heavy spandrels are not required,
6. acoustic surface treatment that can be added easily to interior panel wall at reasonable cost.

A disadvantage of field assembled panels in high humidity environments can be the formation of frost or condensation on the inner liner when insulation is placed only between the subgirt lines. The metal to metal contact (outside sheet-subgirt-inside sheet) should be broken to reduce thermal bridging. A detail which has been used successfully is shown in Fig. 6.1.1. Another option may be to provide rigid insulation between the girt and liner on one side. In any event, the wall should be evaluated for thermal transmittance in accordance with ASHRAE90.1.⁽¹²⁾

6.2 Factory Assembled Panels

Factory assembled panels generally consist of interior liner panels, exterior metal panels and insulation. Panels providing various insulating values are available from several manufacturers. These systems are generally proprietary and must be designed according to manufacturer's recommendations.

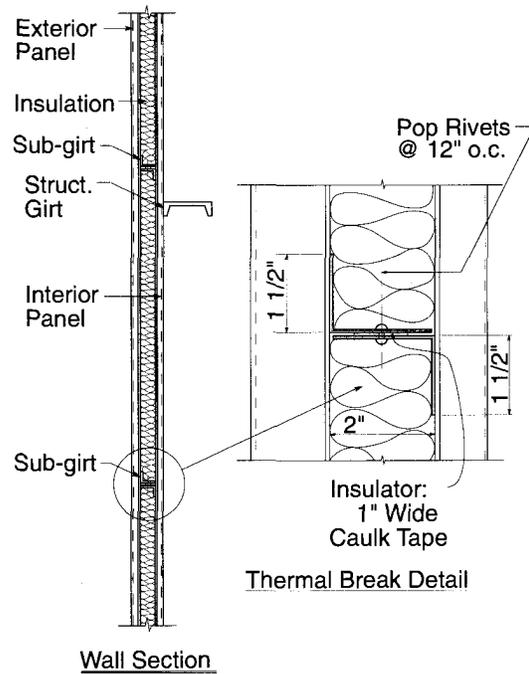


Fig. 6.1.1 Wall Thermal Break Detail

The particular advantages of these factory assembled panels are:

1. Panels are lightweight and require no heavy cranes for erection, no large foundations or heavy spandrels.
2. Panels can have a hard surface interior liner.
3. Panel side lap fasteners are normally concealed producing a "clean" appearance.
4. Documented panel performance characteristics determined by test or experience may be available from manufacturers.

Disadvantages of factory assembled panels include:

1. Once a choice of panel has been made, future expansions may effectively require use of the same panel to match color and profile, thus competition is essentially eliminated.
2. Erection procedures usually require starting in one corner of a structure and proceeding to the next corner. Due to the interlocking nature of the panels it may be difficult to add openings in the wall.
3. Close attention to coordination of details and tolerances with collateral materials is required.
4. Thermal changes in panel shape may be more apparent.

6.3 Precast Wall Panels

Precast wall panels for industrial buildings could utilize one or more of a variety of panel types including:

1. hollow core slabs,
2. double-T sections,
3. site cast tilt-up panels,
4. factory cast panels.

Panels can be either load bearing or nonload bearing and can be obtained in a wide variety of finishes, textures and colors. Also panels may be of sandwich construction and contain rigid insulation between two layers of concrete. Such insulated panels can be composite or non-composite. Composite panels normally have a positive concrete connection between inner and outer concrete layers. These panels are structurally stiff and are good from an erection point of view but the "positive" connection between inner and outer layers may lead to exterior surface cracking when the panels are subjected to a temperature differential. The direct connection can also provide a path for thermal bridging which may be a problem in high humidity situations.

True sandwich panels connect inner and outer concrete layers with flexible metal ties. Insulation is exposed at all panel edges. These panels are more difficult to handle and erect, but normally perform well.

Precast panels have advantages for use in industrial buildings:

1. A hard surface is provided inside and out.
2. These panels produce an architecturally "clean" appearance.
3. Panels have inherent fire resistance characteristics.
4. Intermediate girts are usually not required.
5. Use of load bearing panels can eliminate exterior framing and reduce cost.
6. They provide an excellent sound barrier.

Disadvantages of precast wall panel systems include:

1. Matching colors of panels in future expansion may be difficult.
2. Composite sandwich panels have "cold spots" with potential condensation problems at panel edges.
3. Adding wall openings can be difficult.

4. Panels have poor sound absorption characteristics.
5. Foundations and grade beams may be heavier than for other panel systems.
6. Heavier eave struts are required for steel frame structures than for other systems.
7. Heavy cranes are required for panel erection.
8. If panels are used as load bearing elements, expansion in the future could present problems.
9. Close attention to tolerances and details to coordinate divergent trades are required.
10. Added dead weight of walls can affect seismic design.

6.4 Masonry Walls

Use of masonry walls in industrial buildings is common. Walls can be load bearing or non-load bearing.

Some advantages of the use of masonry construction are:

1. A hard surface is provided inside and out.
2. Masonry walls have inherent fire resistance characteristics.
3. Intermediate girts are usually not required.
4. Use of load bearing walls can eliminate exterior framing and reduce cost.
5. Masonry walls can serve as shear walls to brace columns and resist lateral loads.
6. Walls produce a flat finish, resulting in an ease of both maintenance and dust control considerations.

Disadvantages of masonry include:

1. Masonry has comparatively low material bending resistance. Walls are normally adequate to resist normal wind loads, but interior impact loads can cause damage.
2. Foundations may be heavier than for metal wall panel construction.
3. Special consideration is required in the use of masonry ties, depending on whether the masonry is erected before or after the steel frame.
4. Buildings in seismic regions may require special reinforcing and added dead weight may increase seismic forces.

6.5 Girts

Typical girts for industrial buildings are hot rolled channel sections or cold-formed light gage C or Z sections. In recent years, cold-formed sections have gained popularity because of their low cost. As mentioned earlier, cold-formed Z sections can be easily lapped to achieve continuity resulting in further weight savings and reduced deflections, Z sections also ship economically. Additional advantages of cold-formed sections compared with rolled girt shapes are:

1. Metal wall panels can be attached to cold-formed girts quickly and inexpensively using self-drilling fasteners.
2. The use of sag rods is often not required.

Hot-rolled girts are often used when:

1. Corrosive environments dictate the use of thicker sections.
2. Common cold-formed sections do not have sufficient strength for a given span or load condition.
3. The girts will receive substantial abuse from operations.
4. Designers are unfamiliar with the availability and properties of cold-formed sections.

Both hot-rolled and cold-formed girts subjected to pressure loads are normally considered laterally braced by the wall sheathing. Negative moment regions in continuous girt systems are typically considered laterally braced at inflection points and at girt to column connections. Continuous systems have been analyzed by assuming:

1. a single prismatic section throughout, or
2. a double moment of inertia condition within the lapped section of the cold-formed girt.

Research indicates that an analytical model assuming a single prismatic section is closer to experimentally determined behavior.⁽³⁴⁾

For the design of hot-rolled channels, the beneficial lateral support provided by cladding attached to the tension flange has generally been ignored. The use of sag rods is generally required to maintain horizontal alignment of hot-rolled sections. The sag rods are often assumed to provide lateral restraint against buckling for suction loads. Lateral stability based on this assumption is checked using Chap. F of the AISC Specification.

The typical design procedure for hot-rolled girts is as follows:

1. Select the girt size based on pressure loads, assuming full flange lateral support.
2. Check the selected girt for sag rod requirements based on deflections and bending stresses about the weak axis of the girt.
3. Check the girt for suction loads using Chap. F of the AISC Specification.
4. If the girt is inadequate, increase its size or add sag rods.
5. Check the girt for serviceability requirements.

Cold-formed girts should be designed in accordance with the provisions of the American Iron and Steel Institute (AISI) Specification for the Design of Cold-Formed Steel Structural Members,⁽⁴³⁾ or the AISI Load and Resistance Factor Design Specification for Cold-Formed Steel Structural Members.⁽²¹⁾ Many manufacturers of cold-formed girts have provided this design and offer load span tables to aid design.

Section C3.1.2 "Lateral Buckling Strength" of the AISI Specification provides a means for determining cold-formed girt strength when the compression flange of the girt is attached to sheathing (fully braced) or when discrete point braces (sag rods) are used. For lapped systems, the sum of the moment capacities of the two lapped girts is normally assumed to resist the negative moment over the support. For full continuity to exist, a lap length on each side of the column support should be equal to at least 1.5 times the girt depth.⁽³⁴⁾ Additional provisions are given in Section C3 for strength considerations relative to shear, web crippling, and combined bending and shear.

Section C3.1.3 "Beams with One Flange Attached to Deck or Sheathing" provides a simple procedure to design cold-formed girts subjected to suction loading. The basic equation for the determination of the girt strength is:

$$M_n = R S_e F_y$$

where

- R = 0.40 for simple span C sections.
= 0.50 for simple span Z sections.
= 0.60 for continuous span C sections.
= 0.70 for continuous span Z sections.
- S_e = Elastic section modulus of the effective section calculated with the extreme compression or tension fiber at F_y.
- F_y = Design yield stress as determined in Section A5.2.1 of the Specification.

The procedure does not apply to a continuous girt system between inflection points adjacent to a support.

Other restrictions relative to girt geometry, wall panels, fastening systems between wall panels and girts, etc. are discussed in the AISI Specifications.

6.6 Wind Columns

When bay spacings exceed 30 feet additional intermediate columns may be required to provide for economical girt design. Two considerations that should be emphasized are:

1. Sufficient bracing of the wind columns to accommodate wind suction loads is needed. This is normally accomplished by bracing the interior flanges of the columns with angles which connect to the girts.
2. Proper attention should be paid to the top connections of the columns. For intermediate sidewall columns, secondary roof framing members must be provided to transfer the wind reaction at the top of the column into the roof bracing system. Do not rely on "trickle theory" (i.e. "a force will find a way to trickle out of the structure."). A positive and calculable system is necessary to provide a traceable load path (i.e. Fig. 6.6.1). Bridging systems or bottom chord extension on joists can be used to dissipate these forces, but the stresses in the system must be checked. If the wind columns have not been designed for axial load, a slip connection would be necessary at the top of the column.

Small wind reactions can be transferred from the wind columns into the roof diaphragm system as shown in Fig. 6.6.2.

Allowable values for attaching metal deck to structural members can be obtained from screw manufacturers. Allowable stresses in welds to metal deck can be determined from the American Welding Society Standard, Specification for Welding Sheet Steel in Structures, AWS D1.3,⁽⁴⁴⁾ or from the AISI Specifications.⁽²¹⁾⁽⁴³⁾ In addition to determining the fastener requirements to transfer the concentrated load into the diaphragm, the designer must also check the roof diaphragm for its strength and stiffness. This can be accomplished by using Ref. 11.

7. FRAMING SCHEMES

The selection of "the best" framing scheme for an industrial building without cranes is dependent on numerous considerations, and often depends on the owners requirements. It may not be possible to give a list of rules by which the best such scheme can be assured. If "best" means low initial cost, then the owner may face major expenses in the future for operational expenses or problems

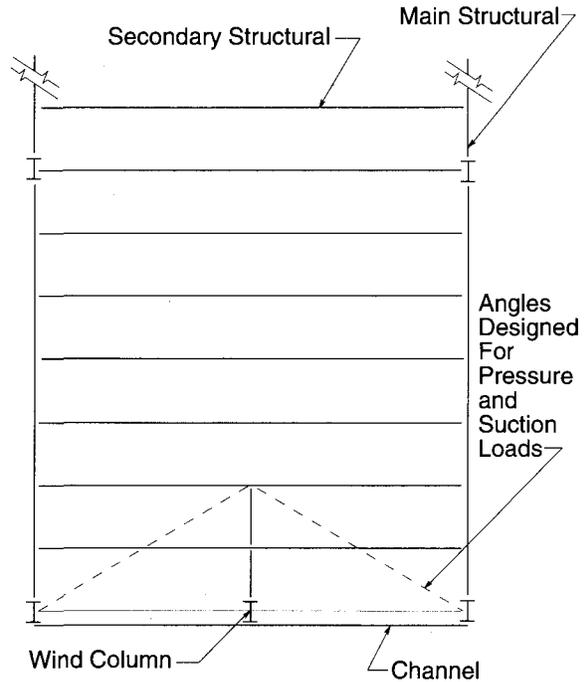


Fig. 6.6.1 Wind Column Reaction Load Transfer

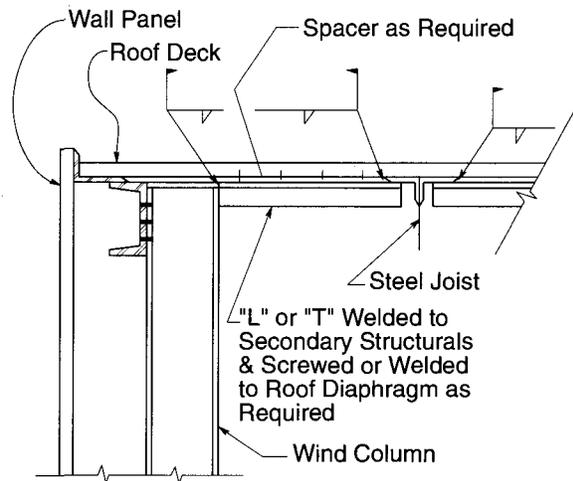


Fig. 6.6.2 Wind Column Reaction Load Transfer

with expansion. Extra dollars invested at the outset reduce potential future costs.

The economics of use of long span vs. short span joists and purlins has been mentioned previously in this guide. This section expands on the selection of the main framing system. No attempt has been made to evaluate foundation costs. In general, if a deep foundation system (e.g., piles or drilled piers) is required, longer bay spacings are normally more economical.

The consideration of bay sizes must include not only roof and frame factors but also the wall system. The cost of large girts and thick wall panels may cancel the savings anticipated if the roof system alone is considered.

AISC offers two manuals that may aid in the design of efficient framing details: Engineering for Steel Construction⁽¹³⁾ and Detailing for Steel Construction.⁽¹⁰⁾

7.1 Braced Frames vs. Rigid Frames

The design of rigid frames is explained in numerous textbooks and professional journals and will not be covered here; however, a few concepts will be presented concerning the selection of a braced versus a rigid frame structural system. There are several situations for which a rigid frame system is likely to be superior.

1. Braced frames may require bracing in both walls and roof. Bracing frequently interferes with plant operations and future expansion. If either consideration is important, a rigid frame structure may be the answer.
2. The bracing of a roof system can be accomplished through X-bracing or a roof diaphragm. In either case the roof becomes a large horizontal beam spanning between the walls or bracing which must transmit the lateral loads to the foundations. For large span to width ratios (greater than 3:1) the bracing requirements become excessive. A building with dimensions of 100 feet by 300 feet with potential future expansion in the long direction may best be suited for rigid frames to minimize or eliminate bracing which would interfere with future changes.

Use of a metal building system requires a strong interaction between the designer and the metal building manufacturer, because of much of the detailing process concerning the design is provided by the manufacturer and the options open to the buyer may reflect the limits of the manufacturer's standard product line and details.

Experience has shown that there are occasions when braced frame construction may prove to be more economical than either standard metal building systems or special rigid frame construction when certain sacrifices on flexibility are accepted.

7.2 Tube Columns vs. W Shapes

The design of columns in industrial buildings includes considerations which do not apply to other types of structures. Interior columns can normally be braced only at the top and bottom, thus square tube columns are desirable due to their equal stiffness about both principal axes. Difficult connections with tube members can be eliminated in single-story frames by placing the beams over the tops of the tubes. Thus a simple to fabricate cap plate detail with bearing stiffeners on the girder web may be designed. Other advantages of tube columns include the fact that they require less paint than equivalent W shapes, and they are pleasing aesthetically.

W shapes may be more economical than tubes for exterior columns for the following reasons:

1. The wall system (girts) may be used to brace the weak axis of the column. It should be noted that a stiffener or brace may be required for the column if the inside column flange is in compression and the girt connection is assumed to provide a braced point in design.
2. Bending moments due to wind loads predominate about one axis.
3. It is easier to frame girt connections to a W shape than to a tube section. Because tubes have no flanges, extra clip angles are required to connect girts.

7.3 Economic Considerations

As previously mentioned, bay sizes and column spacing are often dictated by the function of the building. Economics, however, should also be considered. In general, as bay sizes increase, the weight of the horizontal framing increases. This will mean additional cost unless offset by savings in foundations or erection. Studies have indicated that square or slightly rectangular bays usually result in more economical structures.

The development of the most economical framing scheme for a roof system is a worthy goal. In order to evaluate various framing schemes, a prototype general merchandise structure was analyzed using various spans and component structural elements. The structure was a 240'-0" x 240'-0" building with a 25'-0" eave height. The total roof load was 48 psf, and beams with $F_y = 50$ ksi were used. Plastic analysis and design was used. Columns were tubes with a yield strength of 46 ksi.

Variables in the analysis were:

1. Joist spans: 25, 30, 40, 50 and 60 feet.
2. Girder spans, W sections: 25, 30, 40, 48 and 60 feet.

Cost data were determined from several fabricators. The data did not include sales tax or shipping costs. The study yielded several interesting conclusions for engineers involved in industrial building design.

An examination of the tabular data shows that the most economical framing scheme was the one with beams spanning 30 feet and joists spanning 40 feet.

Another factor that may be important is that for the larger bays (greater than 30 ft) normal girt construction becomes less efficient using C or Z sections without intermediate "wind columns" being added. For the 240' x 240' building being considered wind columns could add \$0.10 per square foot, of roof, to the cost. Interestingly, if the building were 120' x 120', the addition of intermedi-

| JOIST DATA Depth (in.) | | RELATIVE COSTS* | | | | |
|---------------------------|----|-----------------------------------|------|------|------|------|
| | | MAIN FRAMING MEMBERS - W SECTIONS | | | | |
| | | Span (ft.) | | | | |
| Span (ft.) | 25 | 30 | 40 | 48 | 60 | |
| 16 | 25 | 1.10 | 1.10 | 1.25 | 1.31 | 1.53 |
| 18 | 30 | 1.12 | 1.07 | 1.20 | 1.28 | 1.50 |
| 24 | 40 | 1.16 | 1.05 | 1.15 | 1.28 | 1.47 |
| 30 | 50 | 1.22 | 1.18 | 1.20 | 1.30 | 1.54 |
| 32 | 60 | 1.33 | 1.30 | 1.30 | 1.33 | 1.60 |

* Cost included fabrication and erection of primary and secondary framing (no deck). A total gravity load of 48 psf was used in all designs. Uplift and lateral bracing requirements were not included.

ate wind columns would add \$0.20 per square foot because the smaller building has twice the perimeter to area ratio as the larger structure.

Additional economic and design considerations are as follows:

1. When steel joists are used in the roof framing it is generally more economical to span the joists in the long direction of the bay.
2. K series joists are more economical than LH joists; thus an attempt should be made to limit spans to those suitable for K joists.
3. For 30 ft to 40 ft bays, efficient framing may consist of continuous or double cantilevered girders supported by columns in one direction and joists spanning the other direction.
4. If the girders are continuous, plastic design is often used. Connection costs for continuous members may be higher than for cantilever design; however, a plastically designed continuous system will have superior behavior when subjected to pattern load cases. All flat roof systems must be checked to prevent ponding problems. See Section 4.5.
5. Simple-span rolled beams are often substituted for continuous or double-cantilevered girders where spans are short. The simple span beams often have adequate moment capacity. The connections are simple, and the savings from easier erection of such systems may overcome the cost of any additional weight.
6. For large bay dimensions in both directions, a popular system consists of cold-formed or hot-rolled steel purlins or joists spanning 20 ft. to 30 ft. to secondary trusses spanning to the primary

trusses. This framing system is particularly useful when heavily loaded monorails must be hung from the structure. The secondary trusses in conjunction with the main trusses provide excellent support for the monorails.

7. Consideration must be given to future expansion and/or modification, where columns are either moved or eliminated. Such changes can generally be accomplished with greater ease where simple span conditions exist.

8. BRACING SYSTEMS

8.1 Rigid Frame Systems

There are many considerations involved in providing lateral stability to industrial structures. If a rigid frame is used, lateral stability parallel to the frame is provided by the frame. However, for loads perpendicular to the main frames and for wall bearing and "post and beam" construction, lateral bracing is not inherent and must be provided. It is important to re-emphasize that future expansion may dictate the use of a rigid frame or a flexible (movable) bracing scheme.

Since industrial structures are normally light and generally low in profile, wind and seismic forces may be relatively low. Rigid frame action can be easily and safely achieved by providing a properly designed member at a column line. If joists are used as a part of the rigid frame the designer is cautioned on the following points:

1. The design loads (wind, seismic, and continuity) must be given on the structural plans so that the proper design can be provided by the joist manufacturer. The procedure must be used with conscious engineering judgment and full recognition that standard steel joists are designed as simple span members subject to dis-

tributed loads. (See Standard Specifications for Standard Steel Joists and Long Span Joists.)⁽⁴⁰⁾ Bottom chords are normally sized for tension only. The simple attachment of the bottom chord to a column to provide lateral stability will cause gravity load end moments which cannot be ignored. The designer should not try to select member sizes for these bottom chords since each manufacturer's design is unique and proprietary.

2. It is necessary for the designer to provide a well designed connection to both the top and bottom chords to develop the induced moments without causing excessive secondary bending moments in the joist chords.
3. The system must have adequate stiffness to prevent drift related problems such as cracked walls and partitions, broken glass, leaking walls and roofs, and malfunctioning or inoperable overhead doors.

8.2 Roof Bracing Systems

Roof Diaphragms

The most economical roof bracing system is achieved by use of a steel deck diaphragm. The deck is provided as the roofing element and the effective diaphragm is obtained at little additional cost (for extra deck connections). A roof diaphragm used in conjunction with wall X-bracing or a wall diaphragm system is probably the most economical bracing system that can be achieved. Diaphragms are most efficient in relatively square buildings; however, an aspect ratio up to three can be accommodated.

Cold-formed steel diaphragm is analogous to the web of a plate girder. That is, its main function is to resist shear forces. The perimeter members of the diaphragm serve as the "flanges".

The design procedure is quite simple. The basic parameters that control the strength and stiffness of the diaphragm are:

1. profile shape,
2. deck material thickness,
3. span length,
4. the type and spacing of the fastening of the deck to the structural members,
5. the type and spacing of the side lap connectors.

The profile, thickness, and span of the deck are typically based on gravity load requirements. The type of fastening (i.e., welding, screws, and power driven pins)

is often based on the designers or contractors preference. Thus the main design variable is the spacing of the fasteners. The designer calculates the maximum shear per foot of diaphragm and then selects the fastener spacing from the load tables. Load tables are most often based on the requirements set forth in References (11) and (37).

Deflections are calculated and compared with serviceability requirements.

The calculation of flexural deformations is handled in a conventional manner. Shear deformations can be obtained mathematically, using shear deflection equations, if the shear modulus of the formed deck material making up the diaphragm is known. Deflections can also be obtained using empirical equations such as those found in References (11) and (37). In addition, most metal deck manufacturers publish tables in which strength and stiffness (or flexibility) information is presented. In order to illustrate the diaphragm design procedure a design example is presented below. The calculations presented are based on Reference (11).

EXAMPLE 8.2.1: Diaphragm Design

Design the roof diaphragm for the structure shown in Fig. 8.2.1. The eave wind loads are shown in the figure.

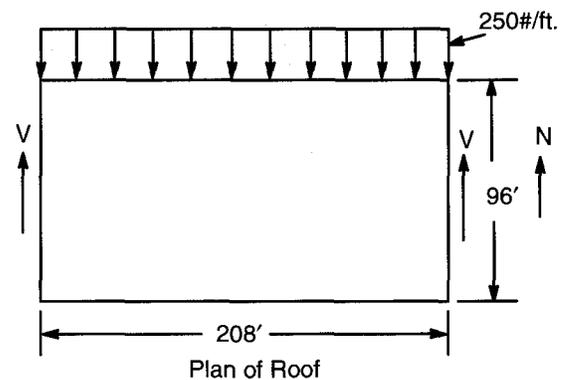


Fig. 8.2.1. Example

Note that the length to width ratio of the diaphragm does not exceed 3, which is the generally accepted maximum for diaphragms.

Assume that a (0.0358 inch thick) intermediate rib deck spanning 5'-6" is used to support the gravity loads. Steel joists span in the north-south direction. Use welds to connect the deck to the structural members and #10 screws for the sidelaps.

Solution:

1. Calculate the maximum diaphragm shear.
 $V = WL/2 = (250)(208)/2 \approx 26,000$ lbs.
 $v = V/96 = 26,000/96 \approx 271$ plf.
2. Obtain the shear capacity of the deck from the SDI Diaphragm Design Manual, Second Edition.

For a 20 gage (.0358" thickness) deck, spanning 5'-6" the allowable shear is:

- (a) 240 plf with a 3/4 weld pattern and one side lap screw.
- (b) 285 plf with a 3/4 weld pattern and two side lap screws.
- (c) 300 plf with a 3/5 weld pattern and one side lap screw.

Use patterns (a) and (b) as shown in Fig. 8.2.1.1:

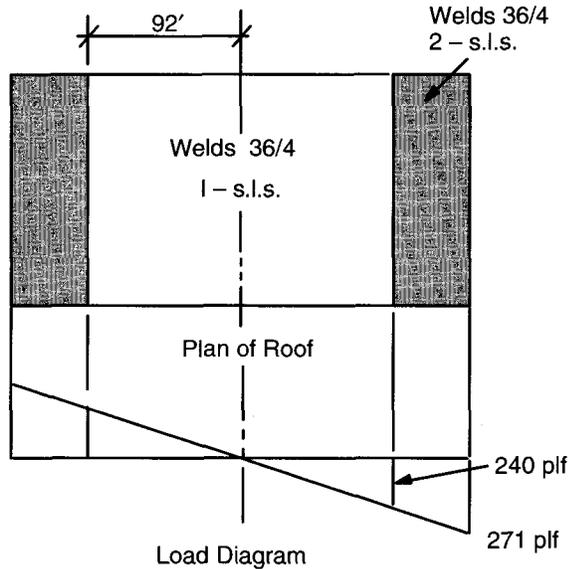


Fig. 8.2.1.1

3. Compute the lateral deflection of the diaphragm.

For simplicity assume one sidelap screw for the entire roof.

The deflection equations are:

(a) For bending: $\Delta_b = \frac{5wL^4}{384EI}$

(b) For shear: $\Delta_s = \frac{wL^2}{8DG'}$

where w = the eave force (kips/ft)
 L = the diaphragm length (ft)
 D = the diaphragm depth (ft)

$$G' = \frac{K2}{3.78 + \left(\frac{.3D_{xx}}{\text{span}} \right) + 3(K1)(\text{span})}$$

From the SDI tables:

$K2 = 1056$

$D_{xx} = D_{ir} = 909$ (intermediate rib)

$K1 = 0.561$ $K1 = .509$ corresponds to 22 ga. deck.

$$\therefore G' = \frac{1056}{3.78 + .3(909) / 5.5 + 3(.561)5.5} = 16.9$$

The moment of inertia, I , can be based on an assumed area of the perimeter member. Assuming the edge member has an area of 3.0 in.², the moment of inertia equals:

$$I = 2Ad^2 = (2)(3.0)(48 \times 12)^2 = 1.99 \times 10^6 \text{ in.}^4$$

The bending deflection equals:

$$\Delta_b = \frac{(5)(.25)(208)^4(1728)}{(384)(29000)(1.99 \times 10^6)} = 0.18 \text{ in.}$$

The shear deflection equals:

$$\Delta_s = \frac{(.25)(208)^2}{(8)(96)(16.9)} = 0.83 \text{ in.}$$

The total deflection equals:

$$\Delta = \Delta_b + \Delta_s = 0.18 + 0.83 = 1.01 \text{ in.}$$

To transfer the shear forces into the east and west walls of the structure the deck can be welded directly to the perimeter beams. The deck must be connected to the perimeter beams with the same number of fasteners as required in the field of the diaphragm. Thus, 5/8 in. dia. arc spot welds 9 inches on center should be specified at the east and west walls.

The reader is cautioned regarding connecting steel deck to the end walls of buildings. If the deck is to be connected to a shear wall and a joist is placed next to the wall, allowance must be made for the camber in the edge joist in order to connect the deck to the wall system. If proper details are not provided, diaphragm connection may not be possible, and field adjustments may be required. Where the edge joist is eliminated near the endwall, the deck can often be pushed down flat on an endwall support. If the joist has significant camber, it may be necessary to provide simple span pieces of deck between the wall and the first joist. A heavier deck thickness may be required due to the loss in continuity. The edge should be covered with a sheet metal cap to protect the roofing materials. This can present an additional problem since the sharp edge of the deck will stick up and possibly damage the roofing.

Along the north and south walls, a diaphragm chord can be provided by attaching an angle to the top of the joists as shown in Fig. 8.2.2. The angle also stiffens the deck edge and prevents tearing of roofing materials along the edge where no parapet is provided under foot traffic. In some designs an edge angle may also be required for the side lap connections for wind forces in the east-west direction. Also, shear connectors may be required to transfer these forces into the perimeter beam. Shown in Fig. 8.2.3 is a typical shear collector.

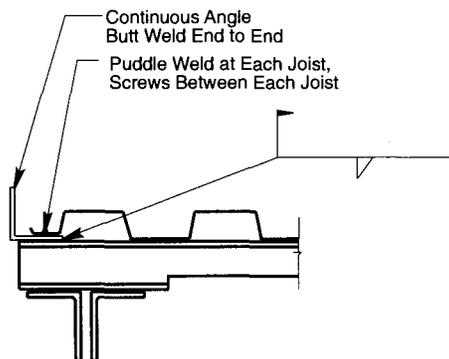


Fig. 8.2.2 Eave Angle

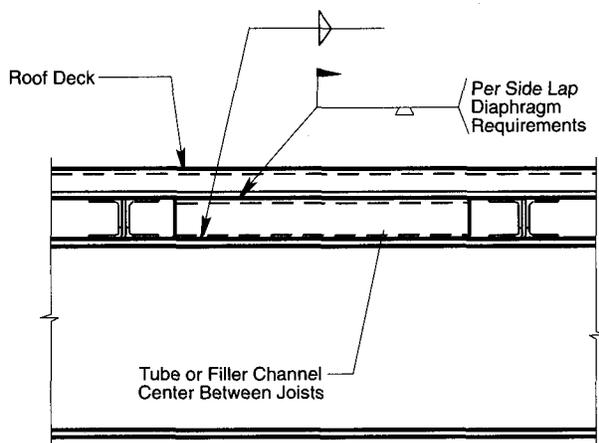


Fig. 8.2.3 Shear Collector

Roof X-Bracing

An alternative to the roof diaphragm is to use X-bracing to develop a horizontal truss system. As with the metal deck diaphragm, as the length to width ratio of the building becomes larger than 3 to 1 the diagonal forces in the truss members may require consideration of an alternate bracing method.

An especially effective way to develop an X-braced roof is to utilize flat bar stock resting on the roof joists. The use of 1/4 in. bar stock does not usually interfere with deck placement and facilitates erection.

8.3 Temporary Bracing

Proper temporary bracing is essential for the timely and safe erection and support of the structural framework until the permanent bracing system is in place. The need for temporary bracing is recognized by the AISC Specifi-

cation^(22,42) (Section M4.2) and by the Code of Standard Practice⁽⁶⁾ (Section 7.9).

The Code of Standard Practice places the responsibility for temporary bracing solely with the erector. This is appropriate since temporary bracing is an essential part of the work of erecting the steel framework.

While the requirements of the Code of Standard Practice are appropriate to establishing the responsibility for erection bracing, two major issues have the potential to be overlooked in the process.

First, the broad standard means that it is difficult to judge the adequacy of temporary bracing in a particular situation, nor is there a codified standard upon which to judge whether or not a minimum level of conformity is met. Secondly, it is not emphasized in the broad standard that the process of erection can induce forces and stresses into components and systems which are not part of the structural steel framework. Unless otherwise specified in the contract documents, it is generally the practice of architects and engineers to design the elements and systems in a building for the forces acting upon the completed structure only.

The lack of clear standards makes it difficult for anyone in the design/construction process to evaluate the performance of the erector relative to bracing without becoming involved in the process itself, which is inconsistent with maintaining the bracing as the sole responsibility of the erector. The lack of emphasis on the need to check the effect of erection forces on other elements allows erection problems to be interpreted as being caused by other reasons. This is most obvious in the erection of steel columns. In order to begin and pursue the erection of a steel framework it is necessary to erect columns first. This means that at one time or another each building column is set in place without framing attached to it in two perpendicular directions. Without such framing the columns must cantilever for a time from the supporting footing or pier unless they are braced by adequate guys or designed as a rigid frame in both directions. The forces induced by the cantilevered column on the pier or footing may not have been considered by the building designer unless this had been specifically requested. It is incumbent upon the steel erector to make a determination of the adequacy of the foundation to support cantilevered columns during erection.

Trial calculations suggest that very large forces can be induced into anchor bolts, piers and footings by relatively small forces acting at or near the tops of columns. Also wind forces can easily be significant, as can be seen in the following example. Fig. 8.3.1 shows a section of unbraced frame consisting of three columns and two beams. The beams are taken as pin ended. Wind forces are acting perpendicular to the frame line.

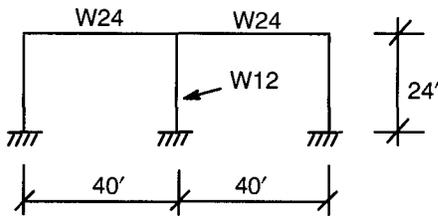


Fig. 8.3.1 Erection Bracing Example

Using a shape factor of 2.0 for a 40 mph wind directed at the webs of the W12 columns, a base moment of approximately 18,000 foot-pounds occurs. If a 5 inch by 5 inch placement pattern were used with four anchor bolts and an ungrouted base plate, a tension force of approximately 21.6 kips would be applied to the two anchor bolts. The allowable force for a 3/4 in A36 anchor bolt is 8.4 kips. Even if the bolts were fully developed (and the 33% increase for wind was utilized) in the concrete, they would be severely overstressed and would likely fail. Four 1-1/8 inch anchor bolts would be required to resist the wind force. Of course not only the size of the anchor bolt is affected, but the design of the base plate and its attachment to the column, the spacing of the anchor bolts and the design of the pier and footing must also be checked.

Guying can also induce forces into the structure in the form of base shears and uplift forces. These forces may not have been provided for in the sizing of the affected members. This must also be checked by the erector. The placement of material such as decking on the incomplete structure can induce unanticipated loadings. This loading must also be considered explicitly. The foregoing points to the need for a well planned bracing scheme prepared and executed by the erector.

Erection bracing involves other issues as well. First, the Code of Standard Practice distinguishes between self-supporting and non-self-supporting structures. The distinction is drawn because the timing of the removal of bracing is affected. In a self-supporting structural steel frame, lateral stability is achieved in the design and detailing of the framework itself. Thus the bracing can be removed when the erector's work is complete. A non-self-supporting steel framework may rely on elements other than the structural steel to provide lateral stability. Such frames should be identified in the contract documents along with the necessary elements to be installed to provide the stability. The coordination of the installation of such elements is a matter which must be addressed between the erector and the owner's agents.

Temporary support beyond the requirements discussed above would be the responsibility of the owner according to the Code of Standard Practice.

The timing of column base grouting affects the performance of column bases during erection. Although the Code of Standard Practice assigns the responsibility for grouting to the owner, the erector should be involved in the coordination of this work.

All of the foregoing points to the need for care, attention and thoroughness on the part of the erector.

9. COLUMN ANCHORAGE

Building columns must be anchored to the foundation system to transfer tension forces, shear forces, and overturning moments. This discussion will be limited to the design of column anchorages for shear and tension forces. The principles discussed here can be applied to the design of anchorages for overturning moments.

Tension forces are typically transferred to the foundation system with anchor bolts. Shear forces are transferred to the foundation system through friction, shear friction, or bearing. Friction should not be considered if seismic conditions exist. Design for these various anchorage methods is addressed in the following text.

Improper design, detailing and installation of anchor bolts have caused numerous structural problems in industrial buildings. These problems include:

1. inadequate sizing of the anchor bolts,
2. inadequate development of the anchor bolts for tension,
3. inadequate design or detailing of the foundation for forces from the anchor bolts,
4. inadequate base plate thickness,
5. inadequate design and/or detailing of the anchor bolt - base plate interface,
6. misalignment or misplacement of the anchor bolts during installation, and
7. fatigue.

The following discussion presents methods of designing and detailing column bases.

9.1 Resisting Tension Forces with Anchor Bolts

The design of anchor bolts for tension consists of four steps:

1. Determine the maximum net uplift for the column.
2. Select the anchor bolt material and number and size of anchor bolts to accommodate this uplift.
3. Determine the appropriate base plate size, thickness and welding to transfer the uplift

forces from the column to the anchor bolts. Refer to AISC Design Guide 1.⁽⁸⁾

4. Determine the method for developing the anchor bolt in the concrete (i.e. transferring the tension force from the anchor bolt to the concrete foundation).

Step 1 - The maximum net uplift for the column is obtained from the structural analysis of the building for the prescribed building loads. The use of light metal roofs on industrial buildings is very popular. As a result of this, the uplift due to wind often exceeds the dead load; thus the supporting columns are subjected to net uplift forces. In addition, columns in rigid bents or braced bays may be subjected to net uplift forces due to overturning.

Step 2 - Anchor bolts are often specified to be ASTM A307 material. However, these bolts are frequently not available (or readily available) for the anchor bolt lengths required. In these instances a threaded rod with a hook or nut is usually used in lieu of an A307 bolt. In anticipation of this, the ASTM A307 specification states that "non-headed anchor bolts, either straight or bent, to be used for structural anchorage purposes, shall conform to the requirements of ASTM Specification A36". In other words, if anchor bolts are designated to be A307 material but are not available for the lengths required, then non-headed anchor bolts made from threaded rods of A36 material with hooks or nuts are to be used. The ASTM A36 specification states that the thread dimensions and nuts to be used with these threaded rods are to conform to the ASTM A307 specification. Consequently, since the ultimate tensile strengths of A36 material and A307 bolts are approximately equal, this substitution results in anchor bolts of approximately equal strength.

Anchor bolts of higher tensile strength than A307 bolts or A36 threaded rods may be used if desired. Table J3.2 in the AISC Specification may be consulted to obtain the allowable stress for the higher strength bolts or threaded rods.

The number of anchor bolts required is a function of the maximum net uplift on the column and the allowable tensile load per bolt for the anchor bolt material chosen. Prying forces in anchor bolts are typically neglected. This is usually justified when the base plate thickness is calculated assuming cantilever bending about the web and/or flange of the column section (as described in Step 3 below). However, calculations have shown that prying forces may not be negligible when the bolts are positioned outside the column profile and the bolt forces are large. A conservative estimate for these prying forces can be obtained using a method similar to that described for hanger connections in the AISC Manual of Steel Construction.

Another consideration in selection and sizing of anchor bolts is fatigue. For most building applications, where uplift loads are generated from wind and seismic forces, fatigue can be neglected because the maximum design wind and seismic loads occur infrequently. However, for anchor bolts used to anchor machinery or equipment where the full design loads may occur more often, fatigue should be considered. In addition, in buildings where crane load cycles are significant, fatigue should also be considered. AISE Technical Report No. 13 for the design of steel mill buildings recommends that 50 percent of the maximum crane lateral loads or side thrust be used for fatigue considerations.

In the past, attempts have been made to pretension or preload anchor bolts to prevent fluctuation of the tensile stress in anchor bolts and, therefore, eliminate fatigue concerns. This is not recommended since creep in the supporting concrete foundation can eventually lead to relaxation of the pretensioning. Table 9.1.1 shows recommended allowable fatigue stresses for non-pretensioned steel bolts. These values are based on S-N data for a variety of different types of bolts. (These data were obtained from correspondence with Professor W. H. Munse of the University of Illinois and are based on results from a number of test studies.) By examining these values, it can be ascertained that, for the AISE loading condition, fatigue will not govern when A36 or A307 anchor bolts are used. However, fatigue can govern the design of higher strength anchor bolts for this load case.

| Number of Loading Cycles ^a | Allowable Tensile Stress (psi) |
|---------------------------------------|--------------------------------|
| 20,000 to 100,000 | 40,000 |
| 100,000 to 500,000 | 25,000 |
| 500,000 to 2,000,000 | 15,000 |
| Over 2,000,000 | 10,000 |

^a — These categories correspond to the loading conditions indicated in Appendix K of the AISC Specification.

Table 9.1.1 Allowable Bolt Fatigue Stress

Step 3 - Base plate thickness may be governed by bending associated with compressive loads or tensile loads. For compressive loads, the design procedure illustrated in the "Column Base Plates" section of Part 3 of the AISC Manual of Steel Construction may be followed. However, for lightly loaded base plates where the dimensions "m" and "n" (as defined in this procedure) are small, thinner base plate thickness can be obtained using yield line theory.

For tensile loads, a simple approach is to assume the anchor bolt loads generate bending moments in the base plate consistent with cantilever action about the web or flanges of the column section (one-way bending). A more refined analysis for bolts positioned inside the column flanges would consider bending about both the web and the column flanges (two-way bending). For the two-way bending approach, the derived bending moments should be consistent with compatibility requirements for deformations in the base plate. In either case, the effective bending width for the base plate can be conservatively approximated using a 45 degree distribution from the centerline of the anchor bolt to the face of the column flange or web. Calculations for required base plate thickness for uplift (tensile) loads are illustrated in Examples 9.4.1 and 9.4.2.

Step 4 - The development of anchor bolts in tension is usually accomplished by using a hook, nut, or steel plate at the embedded end of the anchor bolt. Although chemical bond developed between the anchor bolt and the surrounding concrete may also aid in developing tension in the anchor bolt, it is typically neglected because anchor bolts are often oiled during manufacturing. Appendix B of the ACI Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-85)⁽⁷⁾ recommends that the design for the development of the bolt in concrete accommodate the full tensile strength of the anchor bolt ($A_t \times F_{ut}$) to insure a ductile failure for the anchor. The authors suggest that, for non nuclear structures, the design for the development should accommodate 1.25 times the yield strength of the anchor bolt ($1.25 \times A_t \times F_y$). For an A307 or A36 anchor bolt, this is equivalent to approximately 3/4 of the full or ultimate tensile strength of the bolt. This is consistent with the provisions for development length, splices and mechanical connections listed in Chapter 12 of the ACI Building Code Requirements for Reinforced Concrete Structures (ACI 318-89)⁽⁴⁾. The designer may use his/her judgment in applying this criterion when analysis shows that tension in the anchor bolts is either nonexistent or minimal.

Hooked anchor bolts usually fail by straightening and pulling out of the concrete. This failure is precipitated by a localized bearing failure in the concrete above the hook. Calculation of the development load provided by a hook is illustrated in Example 9.4.1. As indicated in this example, a hook is generally not capable of developing the recommended tensile capacity mentioned in the previous paragraph ($1.25 \times A_t \times F_y$). Therefore, hooks should only be used when tension in the anchor bolt is either nonexistent or minimal.

Tests have shown that a heavy bolt head, or a heavy hex nut on a threaded rod, will develop the full tensile capacity of even high strength anchor bolts when properly embedded and confined in concrete. Therefore, the design for development for headed anchor bolts (typically

threaded rods with heavy hex nuts) is a matter of determining the required embedment depths, edge distances and/or steel reinforcement to prevent failure in the concrete prior to the development of the recommended tensile capacity for the bolt.

As presented in Appendix B of ACI 349-85, failure occurs in the concrete when tensile stresses along the surface of a stress cone surrounding the anchor bolt exceed the tensile strength of the concrete. The extent of this stress cone is a function of the embedment depth, the thickness of the concrete, the spacing between adjacent anchors and the location of adjacent free edges in the concrete. The shapes of these stress cones for a variety of situations are illustrated in Figures 9.1.1, 9.1.2 and 9.1.3. The tensile strength of the concrete (in ultimate strength terms) is represented as a uniform tensile stress of $4\phi\sqrt{f'_c}$ over the surface area of these stress cones. By examining the geometry, it is evident that the ultimate pull-out strength of this cone is equal to $4\phi\sqrt{f'_c}$ times the projected area (A_e) of the cone at the surface of the concrete (excluding the area of the anchor head). Expressions for A_e and the ultimate pullout strength (T_{ULT}) are included in Figures 9.1.1, 9.1.2 and 9.1.3.

The use of plate washers or bearing plates in lieu of or in conjunction with a heavy hex bolt head or nut can increase the surface area of the stress cone and the pullout strength of the concrete for a given embedment depth. However, this may not be the case where bolts are relatively close to the edges of the concrete structure and the extent of the stress cones surrounding the bolts is limited by these edges. In this case, the use of larger washers or bearing plates at the embedded end of the anchor bolts may actually reduce the pullout strength of the concrete (in effect, creating a weakened plane in the concrete). This is often the case with anchor bolts embedded in concrete piers. Appendix B of ACI 349-85 lists dimensional criteria that are to be maintained for washers or bearing plates used as anchor heads. These criteria were developed to be consistent with the dimensional characteristics of a standard heavy hex bolt head or nut.

The previously described stress cone checks rely upon the strength of plain concrete for developing the anchor bolts and would typically apply when columns are supported directly on spread footings, concrete mats or pile caps. However, in some instances the projected area of the stress cones or overlapping stress cones is extremely limited due to edge constraints. Consequently the tensile strength of the anchor bolts cannot be fully developed with plain concrete. This is often the case with concrete piers. In these instances, steel reinforcement in the concrete is used to develop the anchor bolts. This reinforcement often doubles as the reinforcement required to accommodate the tension and/or bending forces in the pier. The reinforcement must be sized and developed for the recommended tensile capacity of the anchor bolts

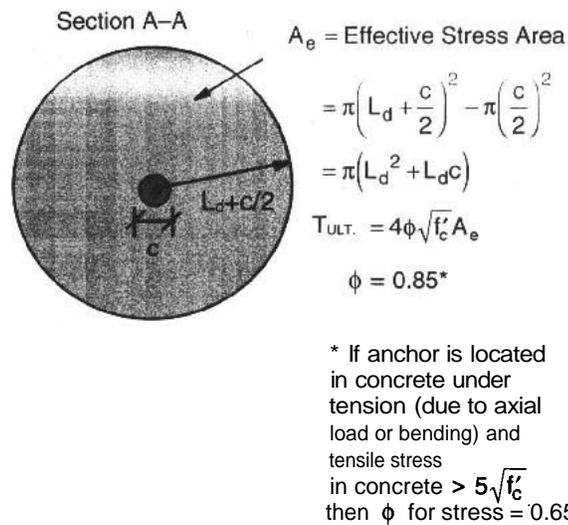
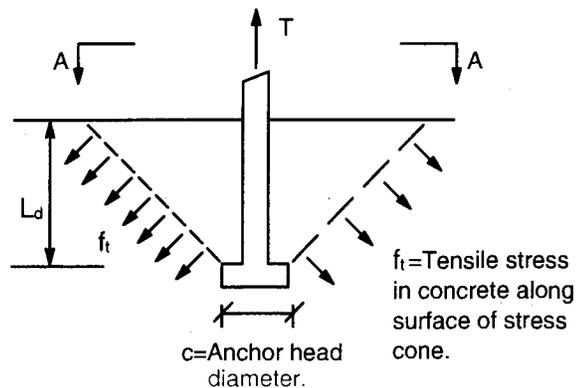
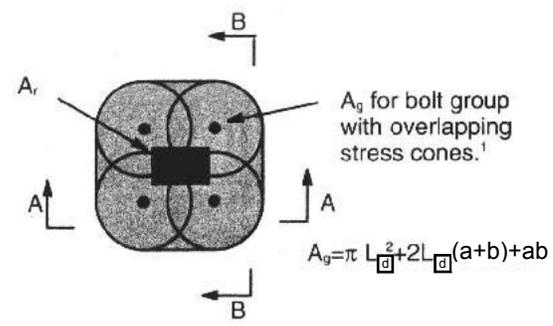


Figure 9.1.1 Stress Cone Development for Single Anchor Bolts in Concrete

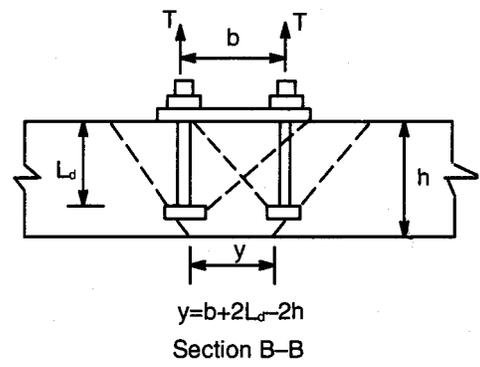
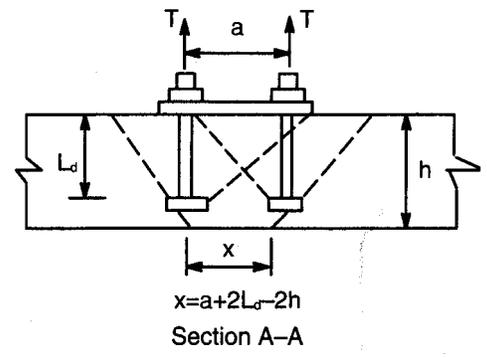
($1.25 \times A_r \times F_y$) on both sides of the potential failure plane described in Figure 9.1.4. The anchor bolt embedment lengths are determined from the required development lengths for this reinforcing steel. These embedment lengths can be reduced by using a larger number of smaller diameter reinforcing bars to develop the anchor bolts. Also, hooks or bends can be added to this reinforcement to minimize development lengths.

Appendix B of ACI 349-85 also lists criteria for minimum concrete side cover (m_s) on anchor bolts to prevent "failure due to lateral bursting forces at the anchor head". These lateral bursting forces are associated with tension in the anchor bolts. The failure plane or surface in this case is assumed to be cone shaped and radiating from the anchor head to the adjacent free edge or side of the concrete structure. This is illustrated in Figure 9.1.5. As with the pullout stress cones, overlapping of the stress cones associated with these lateral bursting forces should be considered. The ultimate tensile strength of the concrete for resisting these bursting forces is equal to $4\phi\sqrt{f'_c}$ over the projected area of these cones at the free edge of the concrete.



$T_{ULT.} = (\text{For Bolt Group}) = 4\phi \sqrt{f'_c} A_e$

$\phi = 0.85$ (See footnote 2)



1. This area is an approximation for the area enclosed by the four overlapping shear cones. The error associated with this approximation is typically small.
2. If anchor is located in concrete under tension (due to axial load or bending) and tensile stress is $> 5\sqrt{f'_c}$, then ϕ for stress cone strength calculation = 0.65

Figure 9.1.2 Stress Cone Limitations Due to Overlapping Stress Cones and Overall Thickness of the Concrete

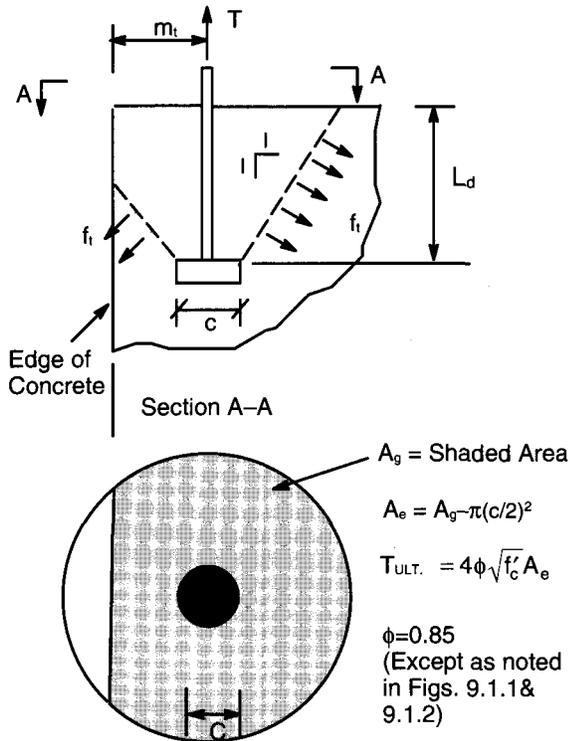


Fig. 9.1.3 Limitations on Tensile Stress Cone for an Anchor Bolt Located Near an Edge

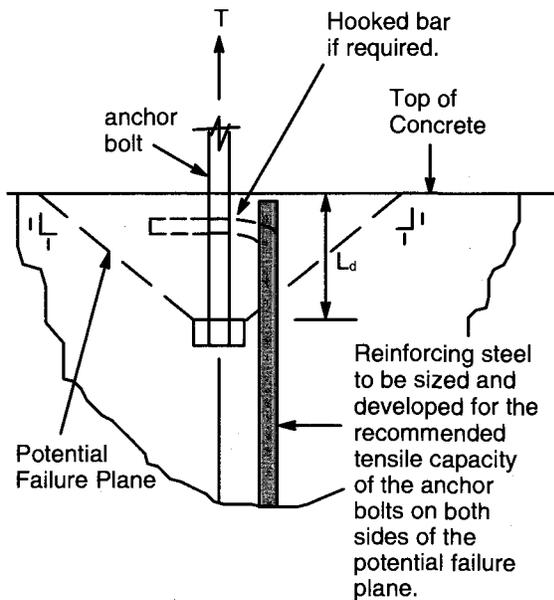


Fig. 9.1.4 The Use of Steel Reinforcement for Developing Anchor Bolts

As a design aid, Table 9.1.2 lists embedment length (L_d) and edge distance requirements (m_t) for single headed anchor bolts without overlapping stress cones i.e., spacing of anchor bolts is greater than $2 \times (L_d + c/2)$ and $(2 \times m_t)$ respectively. In addition, the curves in Figure 9.1.6 show adjustment factors to be applied to these

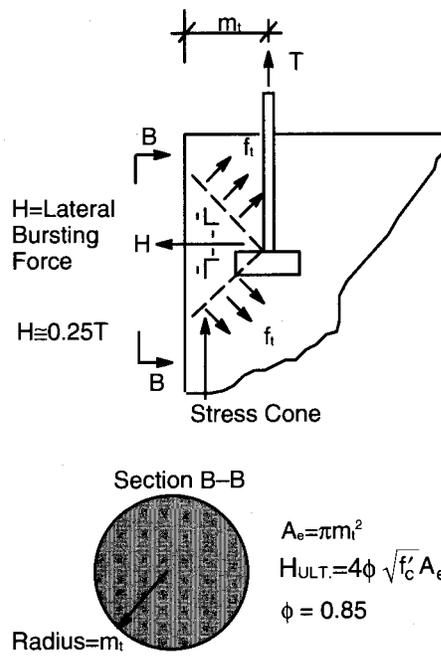


Fig. 9.1.5 Design Check for Lateral Bursting Forces for Anchor Bolts in Tension Located near an Edge

values of L_d and m_t for two bolts with overlapping stress cones. The curves in Figure 9.1.7 show adjustment factors to be applied to L_d for four bolts (arranged in a square pattern) with overlapping stress cones. For this type of bolt arrangement, adjustment factors for m_t are still obtained from Figure 9.1.6. These figures and table do not account for reductions due to lack of thickness in the concrete or edge distance.

Anchor bolt design must be coordinated with the design of the superstructure and the foundations. Calculations for anchor bolt development are illustrated in Example 9.4.1.

9.2 Resisting Shear Forces using Shear Friction Theory

Appendix B of ACI 349-85 describes a "shear-friction" mechanism for transferring shear from anchor bolts to the grout or concrete by bearing near the surface of the concrete. The commentary to ACI 349-85 suggests that this mechanism is developed as follows:

1. Shear is initially transferred through the anchor bolts to the grout or concrete by bearing near the surface of the concrete.
2. This bearing results in the formation of a concrete wedge in front of the anchor bolt approximately one-quarter of the bolt diameter in depth (See Fig. 9.2.1). The formation of this wedge occurs at loads below the shear capacity of the anchor bolts.

| Single A307 & A36 Headed Anchor Bolts or Bolts with Embedded Nuts | | | | | | | | |
|---|-------------------------------|---------------------------------|---------------------------------|---------|-------|------------------|------------------|------------------|
| Bolt Diameter | Tensile Stress Area (A_t) | Heavy Hex Width Across Flat (F) | Effective Diameter (C) (=1.05F) | L_d^* | m_t | m_v $\mu=0.90$ | m_v $\mu=0.70$ | m_v $\mu=0.55$ |
| (in.) | (in. ²) | (in.) | (in.) | (in.) | (in.) | (in.) | (in.) | (in.) |
| 1/2 | 0.142 | 0.875 | 0.92 | 2.9 | 1.7 | 4.4 | 3.9 | 3.5 |
| 5/8 | 0.226 | 1.0625 | 1.12 | 3.6 | 2.1 | 5.6 | 4.9 | 4.4 |
| 3/4 | 0.334 | 1.25 | 1.31 | 4.6 | 2.5 | 6.8 | 6.0 | 5.3 |
| 7/8 | 0.462 | 1.4375 | 1.51 | 5.3 | 3.0 | 8.0 | 7.1 | 6.3 |
| 1 | 0.606 | 1.625 | 1.71 | 6.0 | 3.4 | 9.2 | 8.1 | 7.2 |
| 1-1/4 | 0.969 | 2.00 | 2.10 | 7.7 | 4.3 | 11.6 | 10.2 | 9.1 |
| 1-1/2 | 1.41 | 2.375 | 2.49 | 9.3 | 5.2 | 14.0 | 12.3 | 10.9 |
| 1-3/4 | 1.90 | 2.75 | 2.89 | 10.7 | 6.0 | 16.2 | 14.3 | 12.7 |
| 2 | 2.50 | 3.125 | 3.28 | 12.3 | 6.9 | 18.6 | 16.4 | 14.5 |
| 2-1/4 | 3.25 | 3.50 | 3.68 | 14.3 | 7.9 | 21.2 | 18.7 | 16.6 |
| 2-1/2 | 4.00 | 3.875 | 4.07 | 15.6 | 8.8 | 23.5 | 20.8 | 18.4 |
| 2-3/4 | 4.93 | 4.25 | 4.46 | 17.4 | 9.7 | 26.1 | 23.0 | 20.4 |
| 3 | 5.97 | 4.625 | 4.86 | 19.1 | 10.7 | 28.8 | 25.4 | 22.5 |

* Based on ϕ value = 0.85

Note: L_d , m_t values listed are based on concrete with $f'_c = 3000$ psi

Table 9.1.2 Anchor Bolt Properties and Requirements

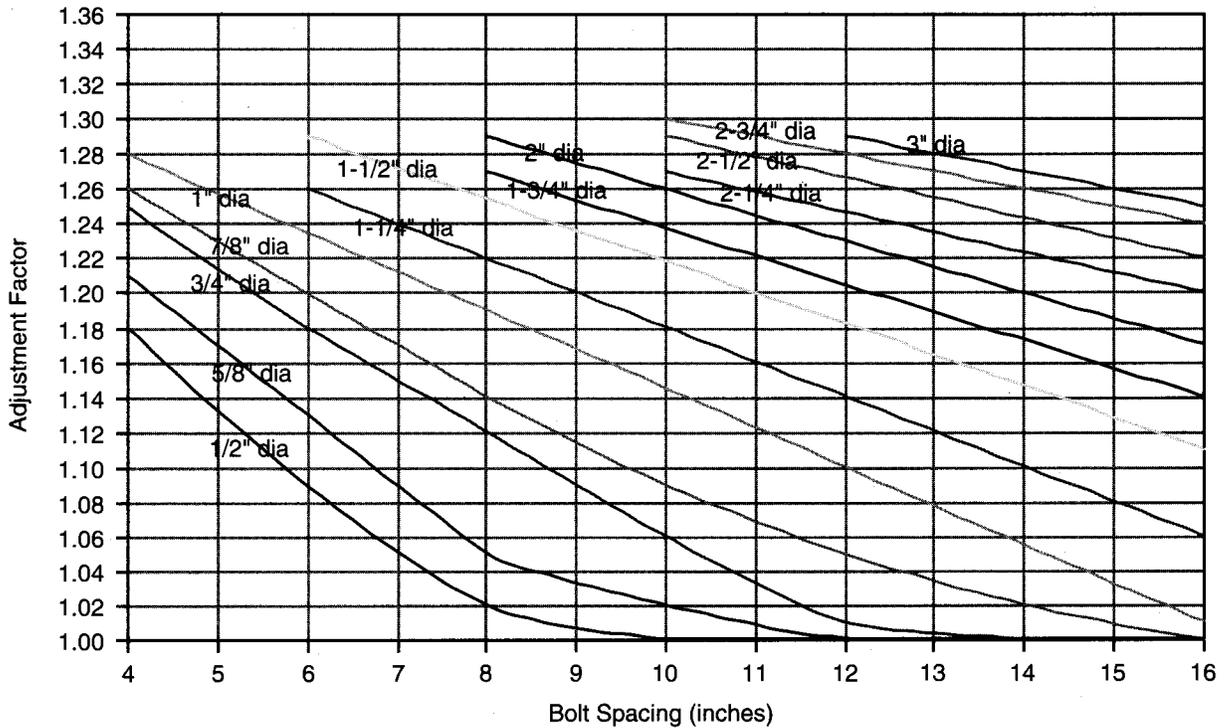


Fig. 9.1.6 Adjustment Factors for Two Anchor Bolts with Overlapping Stress Cones

3. The shear force on this wedge causes the wedge to translate. Lateral translation of the wedge is accompanied by upward movement causing the wedge to push on the bottom of the base plate. The anchor bolts prevent the base plate from moving upward. Therefore, this push results in

tensile forces in the anchor bolts and, in effect, a clamping force between the concrete surface and the bottom of the base plate. Shear transfer at this point is derived from friction between the base plate and the concrete. The relationship for this resistance is described as follows:

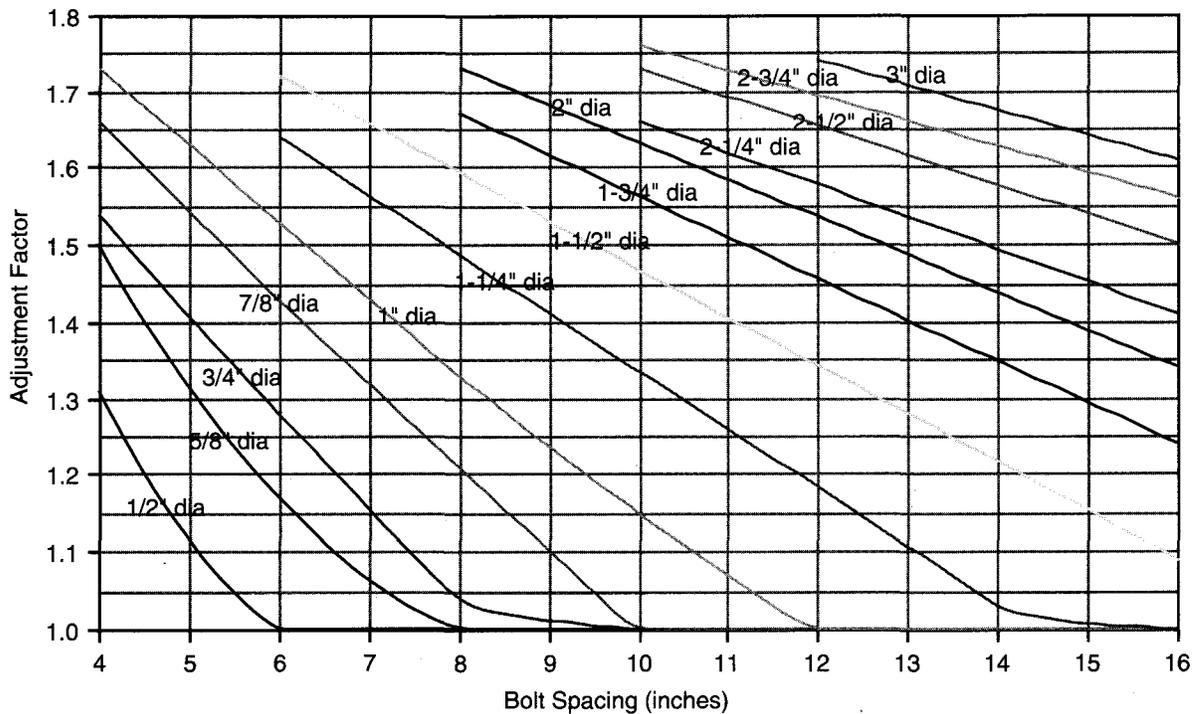


Fig. 9.1.7 Adjustment Factors for Four Anchor Bolts with Overlapping Stress Cones and Square Spacing Pattern

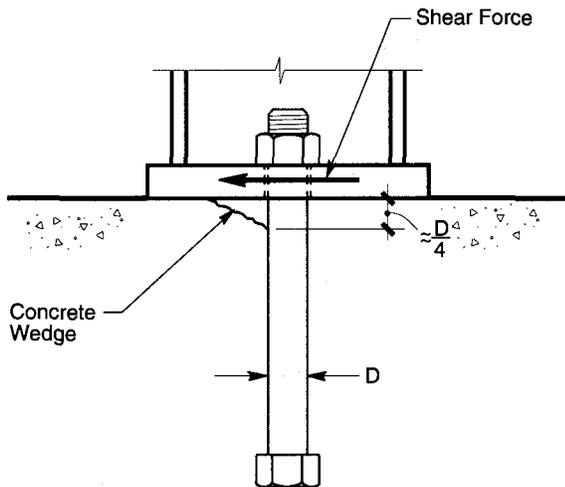


Fig. 9.2.1 Concrete Wedge Formed by Shear Force in Anchor Bolt

$$V_{sf} = N \times \mu$$

where V_{sf} = shear force transferred by shear friction.

N = normal or clamping force

μ = coefficient of friction between the concrete and base plate

Appendix B of ACI 349-85 lists the following coefficients of friction for use with shear friction theory:

- 0.9 for concrete or grout against as-rolled steel with the contact plane a full plate thickness below the concrete surface (i.e., the base plate set into the grout or concrete).
- 0.7 for concrete or grout against as-rolled steel with the contact plane coincidental with the concrete surface.
- 0.55 for grouted conditions with the contact plane between the grout and as-rolled steel exterior to the concrete surface (the normal condition).

The validity of this mechanism has been questioned by some designers. However, according to an article by the principal authors of Appendix B of ACI-349-85⁽⁶⁾, "these design limits have been established using both analytical and test methods" and have been subject to "rigorous evaluation" by ACI Committee 349, the ACI Technical Activities Committee (TAC), and the general membership of the American Concrete Institute.

Using this concept, the design of anchor bolts for shear forces becomes a design for tension with the tensile force in the anchor bolt evaluated as:

$$T_{sf} = V/(n \times \mu)$$

where T_{sf} = tensile force per bolt due to shear friction.

μ = appropriate coefficient of friction.

n = number of bolts.

The location and placement of anchor bolts in concrete is not extremely accurate. In anticipation of this, holes for anchor bolts in column bases are typically oversized. The larger the anchor bolt, the larger the oversize must be. The AISC Manuals list recommended oversize dimensions for these holes in the "Suggested Details" portion of the "Connections" section. In recognition of the oversized holes, it is very possible that all of the anchor bolts are not in bearing with the base plate and, therefore, do not participate fully in transferring shear forces. For a typical four bolt anchor bolt arrangement, a designer may wish to assume that only two of the bolts provide the shear friction resistance. If special plate washers with standard sized holes are added beneath the nuts and welded appropriately to the base plate, all of the anchor bolts could be assumed to provide the shear friction resistance. Special attention should be given to anchor bolts with thick base plates and oversized holes and/or thick unconfined grout beds. These conditions can cause large eccentricities between point of loading and the concrete support, which require bolts to resist bending forces in addition to shear.

All of the criteria previously described for the design of anchor bolts in tension apply to the design for anchor bolts subjected to tension as a result of shear friction. In situations where shear and tension are to be simultaneously transferred at the base of a column, the design tension for the anchor bolt is equal to the sum of the tensile force from uplift (T) and the tensile force due to shear friction (T_{sf}).

For cases where shear and compression exist simultaneously at the base of a column, the compressive force may account for a portion or all of the shear transfer through friction. In this case, the coefficients of friction (μ) referenced earlier may be used with the compressive force to determine the magnitude of this frictional resistance. The shear force in excess of this frictional resistance would be transferred through the anchor bolts using shear friction as previously described. For seismic designs it is recommended that any compression force benefit be neglected.

Appendix B of ACI 349-85 contains a criteria for minimum edge distance (m_v) for bolts when shear friction is used. These criteria are included to prevent localized failure in the concrete adjacent to the bolt. According to the commentary to ACI 349-85, "when the bolt is near an edge, the total shearing force must be developed by tensile stress on a potential failure plane radiating at 45 degrees toward the free edge from the anchor steel at the surface of the concrete". This plane is described by a 45 degree half-cone as illustrated in Figure 9.2.2.

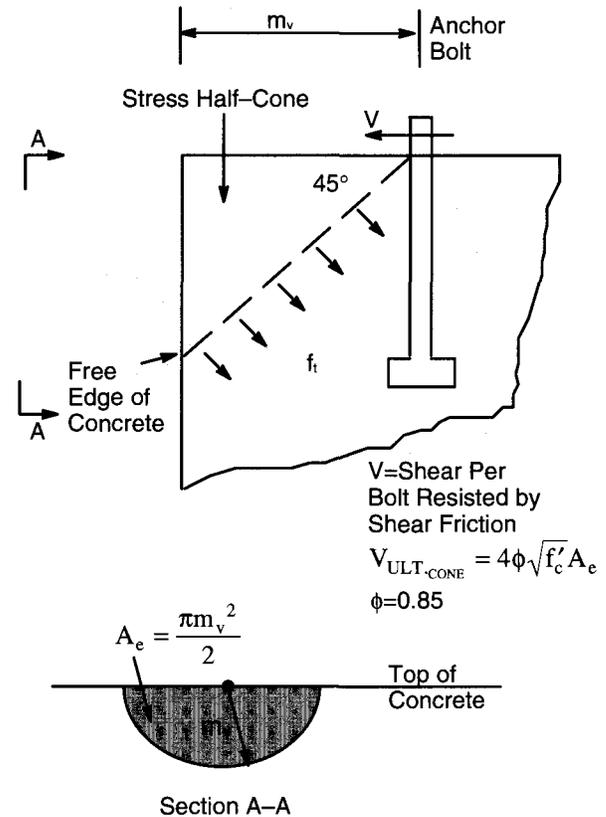


Fig. 9.2.2 Edge Distance Criteria for Shear Forces Resisted by Shear Friction

The ultimate tensile strength of the concrete for resisting this shear force is equal to $4\phi\sqrt{f'_c}$ on the projected area of this half-cone at the free edge. Consideration should be given to overlapping cones for bolts spaced relatively close together. If this edge distance cannot be met, reinforcing similar to that shown in Figure 9.2.3 may be added to prevent this failure. As shown in this figure, the reinforcement is developed on the wedge side of the potential failure plane by attachment to an angle bearing on the surface of the concrete. In many cases it may not be practical to use a normal steel tie or stirrup to reinforce the concrete for this failure mode because the tie or stirrup cannot be adequately developed on the wedge side of the potential failure plane. ACI 349-85 states that even if this reinforcement is added (as shown in Figure 9.2.3), a minimum edge distance of $m_v/3$ must be maintained.

Table 9.1.2 lists values of m_v for a variety of A36 or A307 bolts and the three different values of μ . These values apply to single bolts without overlapping stress cones (i.e. spacing of bolts is greater than $2 \times m_v$). The adjustment factors in Figure 9.1.6 can be used with these values of m_v for two bolts with overlapping stress cones.

ACI 349-85 states that adequate edge distance or reinforcement should be provided to develop the ultimate shear friction capacity of the anchor bolt ($A_t \times F_u \times \mu$). This requirement is to insure ductile failure in the case of

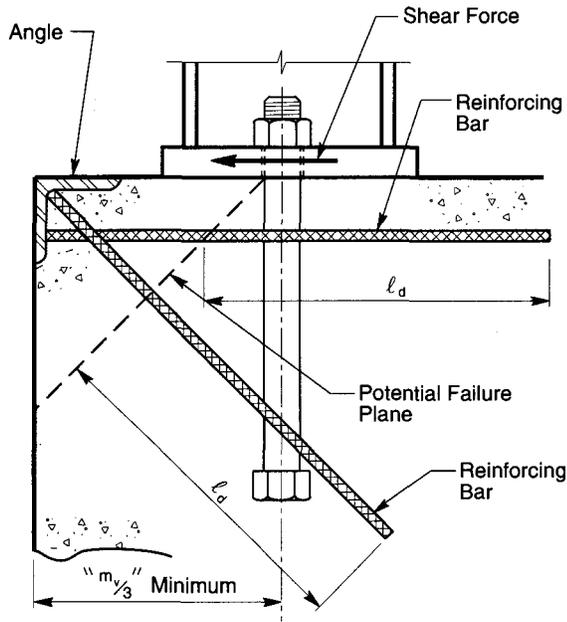


Fig. 9.2.3 Concrete Reinforcement Due to Lack of Edge Distance for Shear

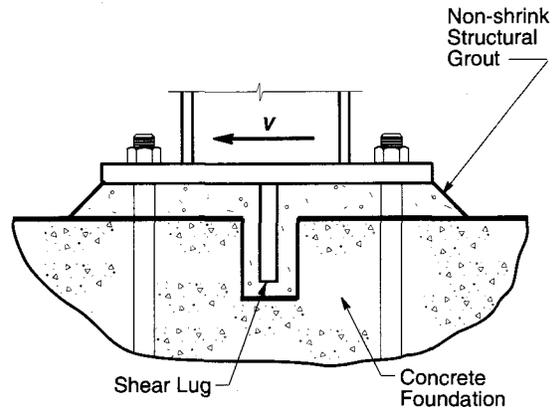
severe shear overload. As discussed previously a more appropriate criterion for non nuclear structures may be to design edge distance or reinforcement to develop a shear friction capacity equal to $(1.25 \times A_t \times F_y \times \mu)$. This is consistent with the previously recommended criteria for development of anchor bolts in tension. The previously mentioned design aids (Table 9.1.2 and Figure 9.1.6) were based on this criterion.

Example 9.4.2 illustrates the design for anchor bolts subjected to combined shear and tension.

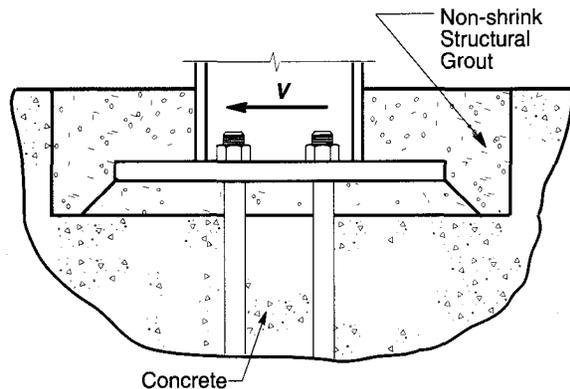
9.3 Resisting Shear Forces through Bearing

Shear forces can be transferred in bearing by the use of shear lugs or by embedding the column in the foundation. These methods are illustrated in Figure 9.3.1. The AISC Steel Design Guide Series, Column Base Plates⁽⁸⁾ discusses design for both of these methods. Additional comments are provided below:

1. For shear lugs or column embedments bearing in the direction of a free edge of the concrete, Appendix B of ACI 349-85 states that in addition to considering bearing failure in the concrete, "the concrete design shear strength for the lug shall be determined based on a uniform tensile stress of $4\phi\sqrt{f'_c}$ acting on an effective stress area defined by projecting a 45 degree plane from the bearing edge of the shear lug to the free surface". The bearing area of the shear lug (or column embedment) is to be excluded from the projected area. This criterion may control or limit the shear capacity of the shear lug or column embedment details in concrete piers.



Shear Lug Detail



Column Embedment Detail

Fig. 9.3.1 Transfer of Base Shears Through Bearing

2. Consideration should be given to bending in the base plate resulting from forces in the shear lug. As a rule of thumb, the authors generally require the base plate to be of equal or greater thickness than the shear lug.
3. Consideration should be given to bending in the column resulting from forces in the shear lug. This can be of special concern when the base shears (most likely due to bracing forces) are large and bending from the force on the shear lug is about the weak axis of the column.
4. Multiple shear lugs may be used to resist large shear forces. Appendix B of ACI 349-85 provides criteria for the design and spacing of multiple shear lugs.

A typical design for a shear lug is illustrated in Example 9.4.3.

A brief discussion on the use of hairpins or tie rods is included to complete the discussion on anchorage design. Hairpins are typically used to incorporate the fric-

tion between the floor slab and the subgrade in resisting the column base shear when individual footings are not capable of resisting horizontal forces. The column base shears are transferred from the anchor bolts to the hairpin through bearing. Problems have occurred when the hairpin bars were placed too low on the anchor bolts (as shown in Figure 9.3.2), thus generating bending in the anchor bolts when the shear friction capacity of the anchor bolt detail is exceeded. This problem can be avoided as shown in Figure 9.3.3 or by providing shear lugs. Since hairpins rely upon the frictional restraint provided by the floor slab, special consideration should be given to the location and type of control and construction joints used in the floor slab to assure no interruption in load transfer, yet still allowing the slab to move.

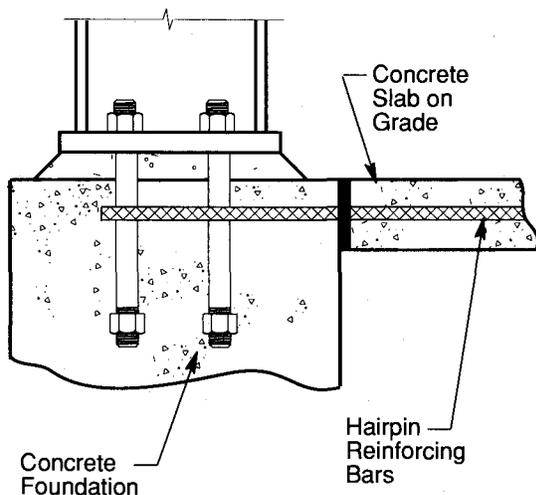


Fig. 9.3.2 Improper Location of Hairpin Bars

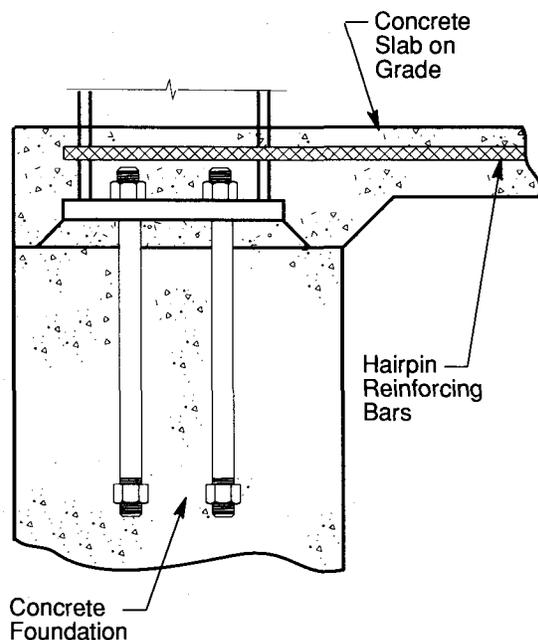


Fig. 9.3.3 Alternate Hairpin Detail

Tie rods are typically used to counteract large shear forces associated with gravity loads on rigid frame structures. When using tie rods with large clear span rigid frames, consideration should be given to elongation of the tie rods and to the impact of these elongations on the frame analysis and design. Again special attention to the dimension between the base plate and the tie rods is required. In addition significant amounts of sagging or bowing should be removed before tie rods are encased or covered, since the tie rods will tend to straighten when tensioned.

9.4 Column Anchorage Examples (Pinned Base)

EXAMPLE 9.4.1: Column Anchorage For Tensile Loads

Design a base plate and anchorage for a W 10x45 column subjected to a net uplift as a result of the following loadings:

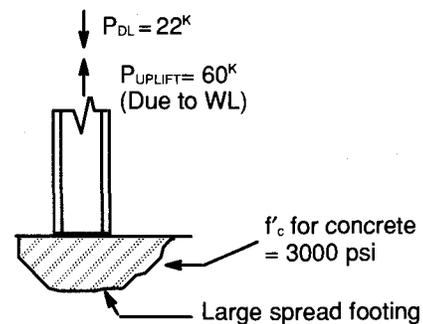


Fig. 9.4.1 Example

Procedure:

1. Determine the maximum net uplift on the column.
2. Select the type and number of anchor bolts.
3. Determine the appropriate base plate thickness and welding to transfer the uplift forces from the column to the anchor bolts.
4. Determine the method for developing the anchor bolts in the concrete.

Solution:

1. Maximum net uplift = $60 - 22 = 38$ kips.
2. Use four anchor bolts (helps to provide stability during erection).

$$T/\text{Bolt} = 38/4 = 9.5 \text{ kips.}$$

Using Table 1-A in the "Connections" Section of the ASD Manual of Steel Construction, select a 3/4 inch diameter A307 bolt.

Determine the allowable force for the wind case.

$$T_{\text{allow.}} = 8.8K(4/3) = 11.7 \text{ kips/bolt}$$

Note: Bolts are positioned inside the column profile and bolt forces are not extremely large; therefore, prying forces are negligible.

3. The bolts are positioned inside the column profile with a 4 in. square pattern. To simplify the analysis, conservatively assume the tensile loads in the anchor bolts generate one-way bending in the base plate about the web of the column. This assumption is illustrated by the bolt load distributions shown in Fig. 9.4.2.

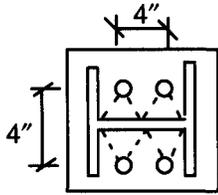


Fig. 9.4.2 Bolt Load Distribution

M_y in the base plate equals the bolt force times the lever arm to the column web face.

$$M_y = 9.5 \left(2 - \frac{0.350}{2} \right) = 17.3 \text{ in.-kips.}$$

The effective width of base plate for resisting M_y at the face of web = b_{eff} .

Assuming a 45 degree distribution for the bolt loads,

$$b_{\text{eff}} = \left(2 - \frac{0.350}{2} \right) (2) = 3.65''$$

$$S_y = \frac{b_{\text{eff}} \times t^2}{6}$$

$$F_{by} = 0.75 F_y (4/3)$$

$$F_y = 36 \text{ ksi}$$

$$F_{by} = 0.75 (36) (4/3) = 36.0 \text{ ksi}$$

$$t_{\text{req'd.}} = \sqrt{\frac{M_y (6)}{b_{\text{eff}} (F_{by})}}$$

$$t_{\text{req'd.}} = \sqrt{\frac{17.3 (6)}{3.65 (36.0)}} = 0.89''$$

Use a 1 in. thick plate ($F_y = 36 \text{ ksi}$).

For welding of the column to the base plate:

$$\text{Maximum weld load} = \frac{T / \text{Bolt}}{b_{\text{eff}}} = \frac{9.5}{3.65} = 2.60 \text{ k / in.}$$

Minimum weld for a 1 inch thick base plate = 5/16 in. (Table J2.4 of ASD Specification).

Allowable weld load per inch for a 5/16" fillet weld with E70 electrode:

$$= (5/16) (.707) (21) (4/3) = 6.19 \text{ k/in.}$$

$2.6 < 6.19$ 5/16" Fillet weld on each side of the column web is o.k.

4. As noted earlier, this column is anchored in the middle of a large spread footing. Therefore, there are no edge constraints on the concrete tensile cones and there is no concern regarding edge distance to prevent lateral bursting.

To insure a ductile failure in the case of overload, design the development for the anchor bolts for $1.25 \times A_s \times F_y$. For 3/4" diameter A307 bolts, this is equal to $1.25 (0.334) (36) = 15.0 \text{ kips/bolt}$.

Try using a 3 inch hook on the embedded end of the anchor bolt to develop the bolt.

Assuming uniform bearing on the hook,

$$\text{hook bearing capacity} = \phi (0.85) f'_c (d) (L')$$

where $\phi = 0.70$

f'_c = concrete compressive strength

d = hook diameter

L' = hook length

$$\begin{aligned} \text{Hook bearing capacity} &= 0.70 (0.85) (3000) (3/4) (3) \\ &= 4020 \text{ lbs.} \\ &= 4.02 \text{ kips} < 15.0 \text{ kips N.G.} \end{aligned}$$

Thus a 3 inch hook is not capable of developing the required tensile force in the bolt.

Therefore, use a heavy hex nut to develop the anchor bolt.

According to Table 9.1.2, the required embedment depth (L_d) for a single 3/4" diameter A307 bolt = 4.6 inches. Since the bolt spacing is less than $2(L_d + c/2)$, the stress cones for the bolts overlap. Using Figure 9.1.7, $(L_d)_{\text{req'd.}} = 4.6'' \times 1.54 = 7.1''$

Use an embedment depth = $L_d = 8''$

This is the depth of concrete embedment to the top of the heavy hex nut.

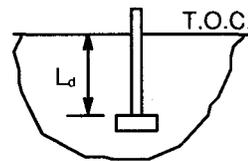


Fig. 9.4.3 Embedment Depth

This solution assumes that the footing has sufficient thickness to allow for the full development of the stress cones (See Figure 9.1.2).

For this to be valid, the footing thickness required

$$\cong L_d + \text{Bolt Spacing} / 2$$

$$= 8'' + 2'' = 10''$$

EXAMPLE 9.4.2: Column Anchorage for Combined Tension and Shear Loads (Pinned Base)

Design a base plate and anchorage for the W 10x45 column examined in Example 9.4.1 but with an additional base shear of 23 kips due to wind. Assume a 2 inch thick grout bed is used beneath the base plate. For this example, the column is assumed to be supported on a 20 inch square pier.

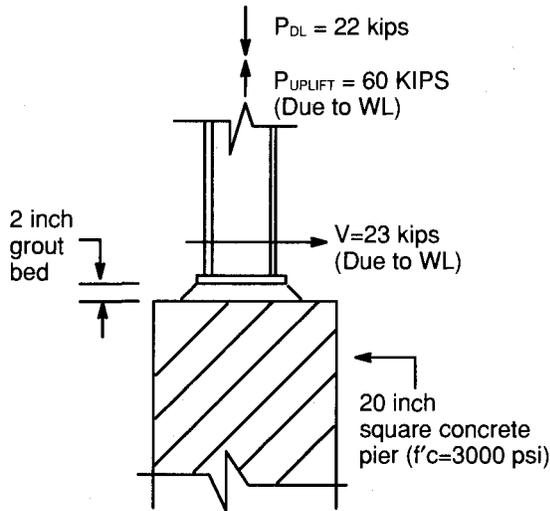


Fig. 9.4.4 Example

Procedure:

1. Determine the maximum net tension in the anchor bolts (due to net tension and shear loads at the base of the column).
2. Select the type and number of anchor bolts.
3. Determine the appropriate base plate thickness and welding to transfer the uplift and shear forces from the column to the anchor bolts.
4. Determine the method for developing the anchor bolts in the concrete.

Solution:

1. As determined in Example 9.4.1, the net uplift on the column = 38 kips. The tension in the bolts due to shear loads (using the shear friction concept) is defined as T_{sf} .

$$T_{sf} = V/(n \times \mu)$$

Use four bolts with plate washers welded to the column base plate to insure that all four bolts participate in transferring the shear load (see discussion in Section 9.2).

Since a grout bed is used beneath the column, $\mu=0.55$.

$$\text{Therefore, } T_{sf} = 23k/(4 \times 0.55) = 10.5 \text{ kips/bolt}$$

$$\begin{aligned} T_{\text{design}} &= T + T_{sf} \\ &= 38/4 + 10.5 \\ &= 20.0 \text{ kips/bolt} \end{aligned}$$

2. As noted in Step 1, a total of (4) anchor bolts are to be used. Using Table 1-A in the "Connections" Section of the AISC Manual of Steel Construction, select 1" diameter, A307 bolts.

$$\begin{aligned} \text{For a wind load case, } T_{\text{allow.}} &= 15.7(4/3) \\ &= 20.9 \text{ kips/bolt} \end{aligned}$$

3. Position the bolts within the profile of the column with a 5 in. square pattern. Again, conservatively assume the tensile loads in the anchor bolts generate one-way bending in the base plate about the web of the column.

$$M_y = 20.9 \left(2.5 - \frac{0.350}{2} \right) = 48.6 \text{ in.-kips}$$

The effective width of the base plate at the face of the web = $b_{\text{eff.}}$.

Assuming a 45 degree distribution for the bolt loads,

$$b_{\text{eff.}} = \left[2.5 - \frac{0.350}{2} \right] (2) = 4.65''$$

$$S_y = \frac{b_{\text{eff}} \times t^2}{6}$$

$$F_{by} = 0.75F_y(4/3)$$

$$F_y = 36 \text{ ksi}$$

$$F_{by} = 0.75(36)(4/3) = 36.0 \text{ ksi}$$

$$t_{\text{req'd.}} = \sqrt{\frac{M_y(6)}{b_{\text{eff}}(F_{by})}}$$

$$t_{\text{req'd.}} = \sqrt{\frac{48.6(6)}{4.65(36.0)}} = 1.32''$$

Use a 1-1/2 in. thick plate ($F_y = 36 \text{ ksi}$).

For welding of the column to the base plate,

Maximum weld load

$$= \frac{T / \text{bolt}}{b_{\text{eff}}} = \frac{20.9}{4.65} = 4.49 \text{ kips / in.}$$

Minimum weld for a 1-1/2 inch thick base plate = 5/16 in.

Allowable weld load for a 5/16" fillet weld with E70 electrode = $0.3125(0.707)(21)(4/3)$ = 6.19 kips/in.

$$4.49 < 6.19 \quad \underline{5/16'' \text{ fillet weld is o.k.}}$$

4. To insure a ductile failure in the case of overload, design the development for the anchor bolts for $1.25 \times A_t \times F_y$.

For 1" diameter A307 bolts, this is equal to:

$$1.25(0.606)(36) = 27.3 \text{ kips/bolt.}$$

As noted earlier, the column is supported on a 20 inch square pier. The maximum tensile load that can be developed by an unreinforced concrete pier is equal to:

$$(4\phi\sqrt{f'_c}) \text{ times the cross sectional area of pier.}$$

$$= 4(0.85)\sqrt{3000}(20)^2$$

$$= 74,500 \text{ lbs.}$$

$$= 74.5 \text{ kips.}$$

The required ultimate tensile strength to insure proper development for the (4) 1" diameter bolts is equal to $4(27.3) = 109.2$ kips.

$$109.2 \text{ kips} > 74.5 \text{ kips.}$$

Therefore, reinforcing steel must be used in the concrete to develop the anchor bolts. For sizing this reinforcement, T/bolt may be taken as $A_t F_y$ (rather than $1.25 \times A_t \times F_y$). This is appropriate because the reinforcing steel is ductile in nature and because the calculated development lengths (per ACI 318) for the reinforcing steel will include the 1.25 factor.

Using grade 60 reinforcing bars,

$$\text{Required } A_{\text{reinforcing}} / \text{bolt} = \frac{(A_t F_y) \text{ anchor bolt}}{\phi F_y}$$

$$= \frac{0.606(36)}{0.9(60)}$$

$$= 0.40 \text{ in.}^2$$

Use one #6 reinforcing bar ($A=0.44 \text{ in.}^2$) with each bolt.

According to Section 12.2 of ACI 318, the basic development length for a #6 bar (grade 60) in 3000 psi concrete = 24.6 inches.

Since the area of steel provided (0.44 in.^2) exceeds the area of steel required (0.40 in.^2), this development length may be reduced by a factor equal to $(0.40/0.44) = 0.91$.

Therefore, the required development length for the #6 bars ($L_{\text{dev.}}$) is calculated as:

$$L_{\text{dev.}} = 0.901(24.6") = 22.5" \quad \text{SAY } 23"$$

An argument can be made that the more conservative lap splice criteria contained in Section 12.15 of ACI 318 should be applied. The authors feel that this is not necessary. The rationale for this decision

is that the Commentary to ACI 318 states that the more conservative criteria applied to lap splices is to "encourage the location of splices away from regions of high tensile stress". It is therefore evident that the more conservative criteria are not associated with reductions in development capabilities within a lapped splice.

The required embedment depth (L_d) for the anchor bolts is determined as shown in Fig. 9.4.5.

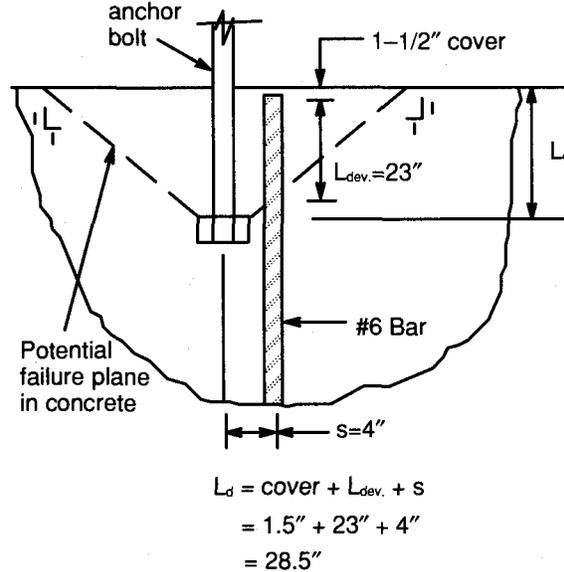


Fig. 9.4.5 Embedment Depth

It is suggested that, when reinforcing steel is added to develop anchor bolts, this reinforcement should be enclosed by tie reinforcement near the top of the reinforcing bars. This reinforcement is added to prevent splitting failures in the concrete prior to yielding the reinforcing steel. The following equation provides the suggested minimum total area of tie reinforcement to be used.

$$(A_{\text{tie}})_{\text{req'd.}} = \frac{T(s)}{2hF_y}$$

where T = sum of yield strengths for the anchor bolts.

s = spacing from anchor bolt to reinforcement,

h = height of ties above top of the anchor head.

F_y = yield strength of the tie reinforcement.

For the example problem,

$$T = 4(0.606)(36) = 87.3 \text{ kips}$$

$$s = 4"$$

$$h = 26" \text{ (assume tie located } 2\text{-}1/2" \text{ from top of pier)}$$

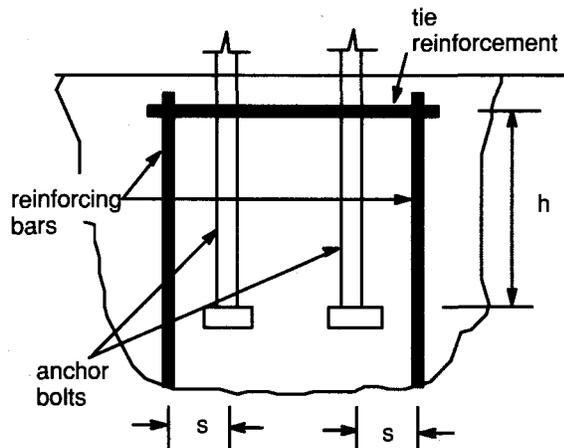


Fig. 9.4.6 Tie Reinforcement

$$F_y = 60 \text{ ksi}$$

$$(A_{tie})_{req'd.} = \frac{87.3(4)}{4(26)(60)} = 0.056 \text{ in.}^2$$

Use a #3 bar.

$$\text{Total tie area provided} = 2(0.11) = 0.22 \text{ in.}^2$$

If the required A_{tie} is large, two ties may be used near the top of the reinforcing bars and "h" taken as the average height of these ties above the top of the anchor heads.

Also, edge distance requirements must be checked. Using Table 9.1.2 and Fig. 9.1.6, the required edge distances are as shown below:

$$m_t = 3.4"(1.26) = 4.3"$$

$$m_v = 7.2"(1.26) = 9.1"$$

For a 20 inch square pier and a 5 inch square anchor bolt pattern, edge distance provided = $(20"-5")/2=7.5"$

Since m_v exceeds 7.5", reinforcement similar to that shown in Figure 9.2.3 should be considered. Alternatively, the pier size could be increased to provide the required edge distance.

EXAMPLE 9.4.3: Design for Shear Lugs (Pinned Base)

Design a shear lug detail for the W 10x45 column considered in Example 9.4.2. See Fig. 9.4.7.

For this detail, the anchor bolts are designed to transfer the net uplift from the column to the pier and the shear lug is designed to transfer the 23 kip shear load to the pier. The design for the anchor bolts is similar to Example 9.4.2 except that tension in the bolts due to shear friction is not included. Therefore, calculations for the anchor

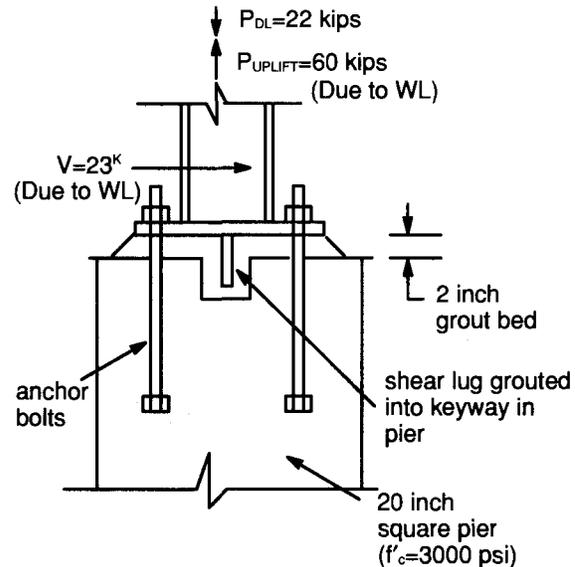


Fig. 9.4.7 Example

bolts are not included in this example. As shown, the anchor bolts are positioned outside the column flanges to prevent interference with the lug detail.

Procedure:

1. Determine the required embedment for the lug into the concrete pier.
2. Determine the appropriate thickness for the lug.
3. Size the welds between the lug and the base plate.

Solution:

1. Two criteria are used to determine the appropriate embedment for the lug. These criteria are the bearing strength of the concrete and the shear strength of the concrete in front of the lug. As discussed in Section 9.3, the shear strength of the concrete in front of the lug is evaluated (in ultimate strength terms) as a uniform tensile stress of $4\phi\sqrt{f'_c}$ acting on an effective stress area defined by projecting a 45 degree plane from the bearing edge of the shear lug to the free surface (the face of the pier). The bearing area of the lug is to be excluded from the projected area. Since this criterion is expressed in ultimate strength terms, the bearing strength of the concrete is also evaluated with an ultimate strength approach. According to ACI 318-89, the ultimate bearing strength of the concrete in contact with the lug is evaluated as:

$$P_u = \phi(0.85f'_cA_1)$$

where P_u = bearing capacity of the concrete in contact with the lug.

$$\phi = 0.70$$

A_1 = embedded area of the shear lug (this does not include the portion of the lug in contact with the grout above the pier).

As discussed in the AISC Design Guide 1⁽⁸⁾, Column Base Plates, it is not recommended that this strength be increased based on confinement from the surrounding concrete, grout and base plate.

The factored shear load = $1.3 \times 23 = 29.9$ kips. (1.3 factor is used since this is a wind load case.)

Equating this to the bearing capacity of the concrete, the following relationship is obtained,

$$0.70(0.85)(3000)(A_1)_{req'd.} = 29900$$

$$(A_1)_{req'd.} = 16.75 \text{ in.}^2$$

Assuming the base plate and shear lug width to be 9 in., the required embedded depth (d) of the lug (in the concrete) is calculated as:

$$d = 16.75/9 = 1.86 \text{ in.} \quad \text{Say } 2".$$

See Figure 9.4.8.

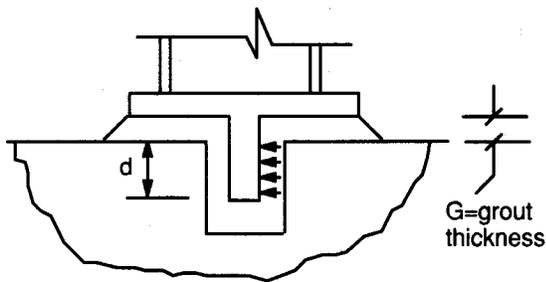


Fig. 9.4.8 Shear Lug Depth

Using this embedment, the shear strength of the concrete in front of the lug is checked. The projected area of the failure plane at the face of the pier is shown in Fig. 9.4.9.

Assuming the lug is positioned in the middle of the pier and the lug is 1 inch thick,

$$a = 5.5" \text{ due to constraints of pier width}$$

$$b = 2" + 9.5" = 11.5"$$

The projected area of this plane (A_v), excluding the area of the lug, is then calculated as:

$$A_v = (5.5 + 9 + 5.5)(11.5) - 2(9) = 212 \text{ in.}^2$$

Using this area, the shear capacity of the concrete in front of the lug (V_u) is calculated as:

$$V_u = 4\phi\sqrt{f'_c}A_v$$

$$= 4(0.85)\sqrt{3000}(212)$$

$$= 39,480 \text{ lbs.} > 29.9 \text{ kips.} \quad \text{o.k.}$$

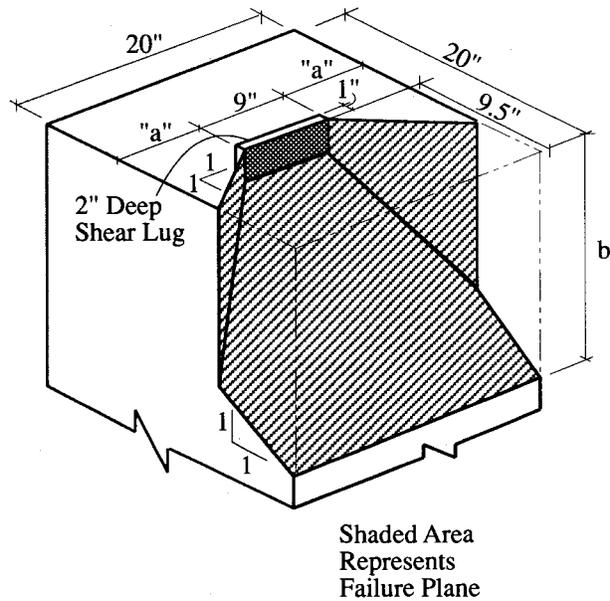


Fig. 9.4.9 Lug Failure Plane

It is concluded that, for a 9 inch wide lug, an embedment depth (d) of 2 inches is adequate.

- Using working loads and a cantilever model for the lug,

$$M_y = V(G + d/2)$$

$$= 23(2 + 2/2) = 69 \text{ kip-in.}$$

Note: G = thickness of grout bed.

$$S_y = bt^2/6 = 9t^2/6$$

$$F_{by} = 0.75F_y(4.3)$$

$$F_y = 36 \text{ ksi}$$

$$t_{req'd.} = \sqrt{\frac{M_y(6)}{9(0.75)(36)(4/3)}}$$

$$= \sqrt{\frac{69(6)}{9(0.75)(36)(4/3)}}$$

$$= 1.13"$$

Use a 1-1/4" thick lug ($F_y = 36$ ksi)

(As discussed in Section 9.3, a minimum thickness of 1-1/4" would also be recommended for sufficient stiffness in the base plate.)

- Most steel fabricators would rather use heavy fillet welds than penetration welds to attach the lug to the base plate. The forces on the welds are as shown in Fig. 9.4.10.

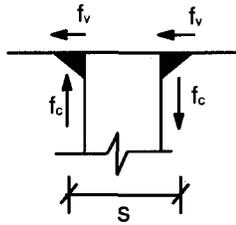


Fig. 9.4.10

Assuming 5/16" fillet welds,

$$s = 1.25 + 0.3125(1/3)(2) = 1.46 \text{ in.}$$

$$f_c = 69/1.46(9) = 5.25 \text{ kips-in.}$$

$$f_v = 23/9(2) = 1.28 \text{ kips-in.}$$

The resultant weld load (f_r) is calculated as:

$$f_r = \sqrt{(5.25)^2 + (1.28)^2} = 5.40 \text{ kips./in.}$$

For a 5/16" fillet weld using E70 electrode, the allowable load (f_{allow}) is calculated as:

$$f_{\text{allow}} = 0.3125(0.707)(21)(4/3)$$

$$= 6.19 \text{ kips/in.} > 5.40 \text{ kips/in.}$$

5/16" fillet welds are o.k.

10. SERVICEABILITY CRITERIA

The design of the lateral load envelope (i.e., the roof bracing and wall support system) must provide for the code imposed loads, which establish the required strength of the structure. A second category of criteria establishes the serviceability limits of the design. These limits are rarely codified and are often selectively applied project by project based on the experience of the parties involved.

In AISC Design Guide No. 3⁽¹⁷⁾ several criteria are given for the control of building drift and wall deflection. These criteria, when used, should be presented to the building owner as they help establish the quality of the completed building.

To be useful, a serviceability criterion must set forth three items: a) loading, b) performance limit, and c) an analysis approach. Concerning lateral forces, the loading recommended by Design Guide No. 3 is the pressure due to wind speeds associated with a ten year recurrence interval. These pressures are approximately 75% of the pressures for strength design criteria, based on a fifty year return period. The establishment of deflection limits is explained below, with criteria given for each of the wall types previously presented. The authors recommend that frame drift be calculated using the bare steel frame only. Likewise the calculations for deflection of

girts would be made using the bare steel section. The contribution of non structural components acting compositely with the structure to limit deflection is often difficult to quantify. Thus the direct approach (neglecting non structural contribution) is recommended and the loads and limits are calibrated to this analysis approach. The deflection limits for the various roof and wall systems are as follows.

10.1 Serviceability Criteria for Roof Design

In addition to meeting strength criteria in the design of the roof structure, it is also necessary to provide for the proper performance of elements and systems attached to the roof, such as roofing, ceilings, hanging equipment, etc. This requires the control of deflections in the roof structure. Various criteria have been published by various organizations. These limits are

1. American Institute of Steel Construction:⁽⁴²⁾
 - a. Depth of fully stressed roof purlins should not be less than ($F_y/1000$) times the span.
2. Steel Deck Institute:⁽³⁸⁾
 - a. Maximum deflection of deck due to uniformly distributed live load: span over 240.
 - b. Maximum deflection of deck due to a 200 lb concentrated load at midspan on a one foot section of deck: span over 240.
3. Steel Joist Institute:⁽⁴⁰⁾
 - a. Maximum deflection of joists supporting plaster ceiling due to design live load: span over 360.
 - b. Maximum deflection of joists supporting ceilings other than plaster ceilings due to design live load: span over 240.
4. National Roofing Contractors Association (NRCA)⁽²⁹⁾:
 - a. Maximum deck deflection due to full uniform load: span over 240.
 - b. Maximum deck deflection due to 300 lb load at midspan: span over 240.
 - c. Maximum roof structure deflection due to total load: span over 240.
5. Factory Mutual:⁽³⁾
 - a. Maximum deck deflection due to a 300 lb concentrated load at midspan: span over 200.

AISC Design Guide No. 3 also presents deflection limits for purlins supporting structural steel roofs (both through fastener types and standing seam types). First, a

limiting deflection of span over 150 for snow loading is recommended. Secondly, attention is drawn to conditions where a flexible purlin parallels nonyielding construction such as at the building eave. In this case deflection should be controlled to maintain positive roof drainage. The appropriate design load is suggested as dead load plus 50 percent of snow load or dead load plus 5 psf live load to check for positive drainage under load.

Mechanical equipment, hanging conveyors, and other roof supported equipment has been found to perform adequately on roofs designed with deflection limits in the range of span over 150 to span over 240 but this criteria should be verified with the equipment manufacturer and building owner. Consideration should also be given to differential deflections and localized loading conditions.

10.2 Metal Wall Panels

Relative to serviceability metal wall panels have two desirable attributes: 1) Their corrugated profiles make them fairly limber for out of plane distortions and 2) their material and fastening scheme are ductile (i.e., distortions and possible yielding do not produce fractures). Also, the material for edge and corner flashing and trim generally allows moment and distortion without failure. Because of this the deflection limits associated with metal panel buildings are relatively generous. They are:

1. Frame deflection (drift) perpendicular to the wall surface of frame: eave height divided by 60 to 100.
2. The deflection of girts and wind columns should be limited to span over 120, unless wall details and wall supported equipment require stricter limits.

10.3 Precast Wall Panels

Non load bearing precast wall panels frequently span from grade to eave as simple span members. Therefore drift does not change the statics of the panel. The limitation on drift in the building frame is established to control the amount of movement in the joint at the base of the panel as the frame drifts. This limit has been proposed to be eave height over 100. A special case exists when precast panels are set atop the perimeter foundations to eliminate a grade wall. The foundation anchorage, the embedment of the panel in the soil and the potential of the floor slab to act as a fulcrum mean that the frame deflections must be analyzed for compatibility with the panel design. It is possible to tune frame drift with panel stresses but this requires interaction between frame designer and panel designer. Usually the design of the frame precedes that of the panel. In this case the frame behavior and panel design criteria should be carefully specified in the construction documents.

10.4 Masonry Walls

Masonry walls may be hollow, grouted, solid, or grouted and reinforced. Masonry itself is a brittle, non-ductile material. Masonry with steel reinforcement has ductile behavior overall but will show evidence of cracking when subjected to loads which stress the masonry in tension. When masonry is attached to a supporting steel framework, deflection of the supports may induce stresses in the masonry. It is rarely feasible to provide sufficient steel (stiffness) to keep the masonry stresses below cracking levels. Thus flexural tension cracking in the masonry is likely and when properly detailed is not a considered a detriment. The correct strategy is to impose reasonable limits on the support movements and detail the masonry to minimize the impact of cracking.

Masonry should be provided with vertical control joints at the building columns and wind columns. This prevents flexural stresses on the exterior face of the wall at these locations from inward wind. Because the top of the wall is generally free to rotate, no special provisions are required there. Most difficult to address is the base of the wall joint. To carry the weight of the wall the base joint must be solid, not caulked. Likewise the mortar in the joints make the base of the wall a fixed condition until the wall cracks.

Frame drift recommendations are set to limit the size of the inevitable crack at the base of the wall. Because reinforced walls can spread the horizontal base cracks over several joints, separate criteria are given for them. If proper base joints are provided, reinforced walls can be considered as having the behavior of precast walls; i.e., simple span elements with pinned bases. In that case the limit for precast wall panels would be applicable. Where wainscot walls are used, consideration must be given to the joint between metal wall panel and masonry wainscot. The relative movements of the two systems in response to wind must be controlled to maintain the integrity of the joint between the two materials.

The recommended limits for the deflection of elements supporting masonry are:

1. Frame deflection (drift) perpendicular to an unreinforced wall should allow no more than a 1/16—inch crack to open in one joint at the base of the wall. The drift allowed by this criterion can be conservatively calculated by relating the wall thickness to the eave height and taking the crack width at the wall face as 1/16-inch and zero at the opposite face.
2. Frame deflection (drift) perpendicular to a reinforced wall is recommended to be eave height over 100.
3. The deflection of wind columns and girts should be limited to span over 240 but not greater than 1.5 inches.

Part 2

INDUSTRIAL BUILDINGS - WITH CRANES

11. INTRODUCTION TO PART 2

This section of the guide deals with crane buildings, and will include coverage of those aspects of industrial buildings peculiar to the existence of overhead and underhung cranes. In that context, the major differences between crane buildings and other industrial buildings is the frequency of loading caused by the cranes. Thus, crane buildings should be classified for design purposes according to the frequency of loading.

Crane building classifications have been established in the AISE Technical Report No. 13⁽²⁰⁾ as classes A, B, C and D. Classifications for cranes have been established by the Crane Manufacturers Association of America (CMAA).⁽³⁹⁾ These designations should not be confused with the building designations as can be gathered from the following descriptions of the two classifications.

11.1 AISE Building Classifications

Class A - are those buildings in which members may experience either 500,000 to 2,000,000 repetitions (Loading Condition 3) or over 2,000,000 repetitions (Loading Condition 4) in the estimated life span of the building of approximately 50 years. Loading condition refers to the fatigue criteria given in Appendix K of the AISC Specifications, LRFD⁽²²⁾ and ASD⁽⁴²⁾. The owner must analyze the service and determine which load condition may apply. It is recommended that the following building types be considered as Class A:

- Batch annealing buildings
- Scrap yards
- Billet yards
- Skull breakers
- Continuous casting buildings
- Slab yards
- Foundries
- Soaking pit buildings
- Mixer building
- Steelmaking buildings
- Mold conditioning buildings
- Stripper buildings
- Scarfig yards
- Other buildings as based on predicted operational requirements

Class B - shall be those buildings in which members may experience a repetition from 100,000 to 500,000 cycles of a specific loading, or 5 to 25 repetitions of such load per day for a life of approximately 50 years (Loading Condition 2).

Class C - shall be those buildings in which members may experience a repetition of from 20,000 to 100,000 cycles of a specific loading during the expected life of a structure, or 1 to 5 repetitions of such load per day for a life of approximately 50 years (Loading Condition 1).

Class D - shall be those buildings in which no member will experience more than 20,000 repetitions of a specific loading during the expected life of a structure.

11.2 CMAA Crane Classifications

The following classifications are taken directly from CMAA.

"70-2 CRANE CLASSIFICATIONS

2.1 Service classes have been established so this specification will enable the purchaser to specify the most economical crane for the installation. Specific requirements are shown for these components where design is influenced by classifications. All classes of cranes are affected by the operating conditions so for the purpose of these definitions it is assumed that the crane will be operating in normal ambient temperatures (0 to 100°F) and normal atmospheric conditions (free from dust, moisture and corrosive fumes).

2.2 CLASS A (STANDBY OR INFREQUENT SERVICE)

This service class covers cranes which may be used in installations such as powerhouses, public utilities, turbine rooms, motor rooms and transformer stations where precise handling of equipment at slow speeds with long, idle period between lifts are required. Capacity loads may be handled for initial installation of equipment and for infrequent maintenance.

2.3 CLASS B (LIGHT SERVICE)

This service covers cranes which may be used in repair shops, light assembly operations, service buildings, light warehousing, etc., where service requirements are light and the speed is slow. Loads may vary from no load to occasional full rated loads with two to five lifts per hour, averaging 10 feet per lift.

2.4 CLASS C (MODERATE SERVICE)

This service covers cranes which may be used in machine shops or papermill machine rooms, etc., where service requirements are moderate. In this type of

service the crane will handle loads which average 50 percent of the rated capacity with 5 to 10 lifts per hour, averaging 15 feet, not over 50 percent of the lift at rated capacity.

2.5 CLASS D (HEAVY SERVICE)

This service covers cranes which may be used in heavy machine shops, foundries, fabricating plants, steel warehouses, container yards, lumber mills, etc., and standard duty bucket and magnet operations where heavy duty production is required. In this type of service, loads approaching 50 percent of the rated capacity will be handled constantly during the working period. High speeds are desirable for this type of service with 10 to 20 lifts per hour averaging 15 feet, not over 65 percent of the lifts at rated capacity.

2.6 CLASS E (SEVERE SERVICE)

This type of service requires a crane capable of handling loads approaching a rated capacity throughout its life. Applications may include magnet/bucket combination cranes for scrap yards, cement mills, lumber mills, fertilizer plants, container handling, etc., with twenty or more lifts per hour at or near the rated capacity.

2.7 CLASS F (CONTINUOUS SEVERE SERVICE)

This type of service requires a crane capable of handling loads approaching rated capacity continuously under severe service conditions throughout its life. Applications may include custom designed specialty cranes essential to performing the critical work tasks affecting the total production facility. These cranes must provide the highest reliability with special attention to ease of maintenance features."

The class of crane, the type of crane, and loadings all affect the design. The fatigue associated with crane class is especially critical for the design of crane runways and connections of crane runway beams to columns. Classes E and F produce particularly severe fatigue conditions. The determination of stress levels and load conditions is discussed in more detail in the next section.

The CMAA crane classifications do not relate directly to the AISC loading conditions for fatigue. Based on the average number of lifts for each CMAA crane classification, the crane classes corresponding to the AISC loading conditions are shown in Table 11.2.1.

The MBMA Low Rise Building Systems Manual⁽²⁴⁾ also relates CMAA crane classifications to the AISC loading conditions. The MBMA relationship shown in Table 11.2.2, is

| CMAA Crane Classification | AISC Loading Condition |
|---------------------------|------------------------|
| A, B | 1 |
| C, D | 2 |
| E | 3 |
| F | 4 |

Table 11.2.1 Crane Loading Conditions

based on equations which further define loading conditions by the total weight and lifted load of the crane.

| Service Class | AISC Loading Condition | |
|---------------|------------------------|--------|
| | R ≤ .5 | R > .5 |
| B | – | 1 |
| C | 1 | 2 |
| D | 2 | 3 |

Table 11.2.2 MBMA Crane Service Classes

where

R = TW/(TW + RC) for underhung monorail cranes.

R = TW/(TW + 2RC) for bridge cranes.

TW = The entire crane weight.

RC = The rated capacity of the crane.

The MBMA procedure is recommended for classes B, C, and D since the weight of the cranes can have a very detrimental effect on the fatigue of a runway system.

12. FATIGUE

Crane buildings are often loaded to full design loads, and in many cases the design load will occur thousands of times. Thus design stresses must be selected with regard to fatigue limits.

An eventual failure due to repeated loading and unloading (even if the yield point of the material is never exceeded) is known as fatigue. Fatigue may be observed even if all conditions are near ideal; i.e., the material exhibits excellent notch toughness, no stress concentrations from holes or notches, uniaxial stress condition, ductile microstructure, no residual stress, etc. However, the existence of multistress conditions, conditions affecting ductility, residual stresses, etc. all reduce the fatigue life of a structure. By knowing the maxi-

imum number of cycles to which a structure is subjected, along with the stress ratio, the fatigue strength can be determined.

Contained in Appendix K of the AISC Specifications are guidelines for the design of structures subjected to a cyclic stress variation (fatigue). In Appendix K, allowable stress ranges are given for various geometrical conditions depending on the loading condition (expected number of stress cycles).

Fatigue failures have been observed in many crane buildings. Failures generally occur in crane runway beams and can be associated with an attachment to a beam, such as a stiffener, or they may be due to improper detailing of connections between runway beams or at a runway beam and column. Recommended design procedures and details illustrated in the section on plate girders are to help prevent such failures.

13. CRANE INDUCED LOADS AND LOAD COMBINATIONS

It is recommended that the designer shows, on the drawings, the crane wheel loads, wheel spacing, bumper forces, and the design criteria used to design the structure.

Although loading conditions for gravity, wind, and seismic loads are well defined among building codes and standards, crane loading conditions generally are not.

As mentioned previously, crane fatigue loadings are primarily a function of the class of service, which in turn is based primarily on the number of cycles of a specific loading case. This classification should be based on the estimated life span, rate of loading, and the number of load repetitions, the owner should specify or approve the classification for all portions of a building. A maximum life span of 50 years is generally accepted.

The provisions of the AISC and AISE on crane runway loads are summarized in the following discussion. As an alternate the MBMA Low Rise Building Systems Manual⁽²⁴⁾ provides a comprehensive discussion on crane loads.

13.1 Vertical Impact

AISC and AISE

The allowances for vertical impact are specified as follows: For traveling cab operated cranes not less than 25% of maximum crane wheel loads. The AISC Specifications further indicate that for pendant operated traveling crane support girders and their connections this load may be reduced to 10% of maximum crane wheel loads. The AISE Technical Report No. 13 document requires the use of 20% of the maximum

crane wheel loads for motor room maintenance cranes, with additional requirements for other cranes.

The AISE Report requires impact to be considered in crane columns when one crane is the governing criterion. The AISC specification does not require this. In all cases, impact loading should be considered in the design of column brackets.

13.2 Side Thrust

Horizontal forces exist in crane loadings due to a number of factors including:

1. Runway misalignment
2. Crane skew
3. Trolley acceleration
4. Trolley braking
5. Crane steering

AISC

The total lateral force on crane runways shall be not less than 20% of the sum of the weights of the lifted load and crane trolley. The force shall be applied to the top of the rail and normal to the rail direction and shall be distributed with due regard to the lateral stiffness of the structure supporting the crane rails.

AISE

The AISE document requires that "The recommended total side thrust shall be distributed with due regard for the lateral stiffness of the structures supporting the rails and shall be the greatest of:

- (1) That specified in Table 1 [Shown here as Table 13.2.1].
- (2) 20% of the combined weight of the lifted load, trolley and other lifting devices (i.e. spreader beam, hook, block, rotating mechanism etc.) For stacker cranes this factor shall be 40% of the combined weight of the lifted load, trolley, rigid arm and material handling device.
- (3) 10% of the combined total weight of the lifted load and the crane weight. For stacker cranes this factor shall be 15% of the combined total weight of the lifted load and the crane weight."

AISE requires that radio-operated cranes be considered as cab-operated cranes with regard to side thrusts.

13.3 Longitudinal or Tractive Force

AISC

The longitudinal force, unless specified otherwise, shall be taken as not less than 10% of the maximum wheel loads of the crane applied at the top of the rail.

AISE

The tractive force shall be 20% of the maximum load on driving wheels.

Table 13.3.1 is provided to illustrate the variation between the AISC Specification and AISE Technical Report No. 13 for a particular crane size.

| Crane Type | Total side thrust percent of lifted load |
|--|--|
| Mill crane | 40 |
| Ladle cranes | 40 |
| Clamshell bucket and magnet cranes (including slab and billet yard cranes) | 100 |
| Soaking pit cranes | 100 |
| Stripping cranes | 100 [†] |
| Motor room maintenance cranes, etc. | 30 |
| Stacker cranes (cab-operated) | 200 |
| [†] ingot and mold | |

Table 13.2.1 AISE Crane Side Thrusts

| SIDE THRUST COMPARISON FOR VARIOUS SPECIFICATIONS (100T MILL CRANE) (TROLLEY WT=60000#, ENTIRE WT=157200#) | | |
|---|---|-------------------|
| SPECIFICATION | EQUATION | FORCE TO ONE SIDE |
| AISC | $\frac{0.20}{2}$ (Trolley Wt + Lifted Load) | 26.00 kips |
| AISE | (1) 0.10 (Trolley Wt + Lifted Load) | 26.00 kips |
| | (2) 0.05 (Entire Crane Wt + Lifted Load) | 17.86 kips |
| | (3) 0.20 (Lifted Load) | 40.00 kips |

Table 13.3.1 AISC/AISE Side Thrust Comparison

13.4 Crane Stop Forces

The magnitude of the bumper force is dependent on the energy absorbing device used in the crane bumper. The device may be linear such as a coil spring or nonlinear such as hydraulic bumpers. See Section 18.6 for additional information on the design of the runway stop.

The crane stop, crane bracing, and all members and their connections that transfer the bumper force to the ground, should be designed for the bumper force. It is recommended that the designer indicate on the structural drawings the magnitude of the bumper force assumed in the design. The bump-

er force is generally specified by the owner or crane supplier. If no information can be provided at the time of design Section 6.6 of the MBMA Manual⁽²⁴⁾ can provide some guidance.

13.5 Eccentricities

The bending of the column due to eccentricity of the crane girder on the column seat must be investigated. The critical bending for this case may occur when the crane is not centered over the column but located just to one side as illustrated in Figure 13.5.1. Additional consideration for other eccentricities is discussed in Sections 17.2 and 18.2.

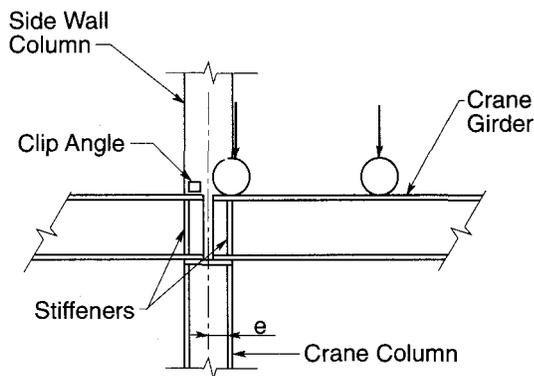


Fig. 13.5.1 Possible Critical Crane Location

13.6 Seismic Loads

Although cranes do not induce seismic loads to a structure, the crane weight should be considered in seismic load determination. The designer should carefully evaluate the number of cranes to be considered in the seismic force determination and their location within the building.

13.7 Load Combinations

In addition to local building codes, the owner may require conformance with AISC or AISE rules. However, in the absence of such rules, the designer should consider the usage of the structure in determining the criteria for the design. Building codes generally may not contain information on how to combine the various crane loads; i.e. which crane loads, and how many cranes should be considered loaded at one time, but generally they do address how crane loadings should be combined with wind, snow, live, seismic, and other loads.

For one crane, each span must be designed for the most severe loading with the crane in the worst position for each element that is affected. As mentioned, when more than one crane is involved in making a lift, most codes are silent on a defined procedure. Engineering judgment on the specific application must be used.

AISE Technical Report No. 13 includes the following provisions for the design of members subject to multiple crane lifts. These provisions are to be used in the design of the supporting elements.

The design of members (and/or frames), connection material and fasteners shall be based on whichever one of the three cases listed hereinafter may govern. Moments and shears for each type loading shall be listed separately (i.e., dead load, live load, crane eccentricity, crane thrust, wind, etc.). The permissible stress range under repeated loads shall be based on fatigue considerations with the estimated number

of load repetitions in accordance with the Building Classification. The owner shall designate an increase in the estimated number of load repetitions for any portion of the building structure for which the projected work load or possible change in building usage warrants.

Case 1. This case applies to load combinations for members designed for repeated loads. The stress range shall be based on one crane (in one aisle only - where aisle represents the zone of travel of a crane parallel to its runway beams) including full vertical impact, eccentric effects and 50% of the side thrust. The number of load repetitions used as a basis for the design shall be 500,000 to 2,000,000 (AISC Loading Condition 3) or over 2,000,000 (AISC Loading Condition 4), as determined by the owner, for Class A construction. Class B and Class C constructions shall be designed for 100,000 to 500,000 (AISC Load Condition 2) and 20,000 to 100,000 (AISC Loading Condition 1) respectively. This case does not apply to Class D buildings. Permissible stress range shall be in accordance with the AISC recommendations (AISC Appendix K).

Case 2. All dead and live loads, including roof live loads, plus maximum side thrust of one crane or more than one crane if specific conditions warrant, longitudinal traction from one crane, plus all eccentric effects and one of the following vertical crane loadings:

1. Vertical load from one crane including full impact.
2. Vertical load induced by as many cranes as may be positioned to affect the member under consideration, not including impact.

Full allowable stresses may be used with no reduction for fatigue. This case applies to all classes of building construction.

Case 3. All dead and live loads including impact from one crane plus one of the following:

1. Full wind with no side thrust but with one crane positioned for maximum vertical load effects.
2. Fifty percent of full wind load with maximum side thrust and vertical load effects from one crane.
3. Full wind with no live load or crane load.
4. Bumper impact at end of runway from one crane.
5. Seismic effects resulting from dead loads of all cranes parked in each aisle positioned for maximum seismic effects.

For Case 3 permissible stresses may be increased 33-1/3 percent. This case applies to all classes of building construction.

Because the standard AISE building classifications were based upon the most frequently encountered situations, they should be used with engineering judgment. The engineer, in consultation with the owner, should establish the specific criteria. For example, other load combinations that have been used by engineers include:

1. A maximum of two cranes coupled together with maximum wheel loads, 50% of the specified side thrust from each crane, and 90% of the specified traction. No vertical impact.
2. One crane in the aisle and one in an adjacent aisle with maximum wheel loads, specified vertical impact, and with 50% combined specified side thrust and specified traction from each crane.
3. A maximum of two cranes in one aisle and one or two cranes in an adjacent aisle with maximum wheel loads, and 50% of the specified side thrust of the cranes in the aisle producing the maximum side thrust, with no side thrust from cranes in the adjacent aisle. No vertical impact or traction.

Additional information relative to loading combinations are contained in the MBMA Low Rise Building Systems Manual. The crane combinations contained in the Manual agree very closely with the AISE combinations.

14. ROOF SYSTEMS

The inclusion of cranes in an industrial building will generally not affect the basic roof covering system. Crane buildings will "move" and any aspect in the roof system that might be affected by such a movement must be carefully evaluated. This generally means close examination of details (e.g. flashings, joints, etc.).

A significant difference in the roof support system design for crane buildings as opposed to industrial buildings without cranes is that (except for very light duty-lightweight cranes) the use of diaphragm roof bracing is not recommended. Whereas wind loads apply rather uniformly distributed forces to the diaphragm, cranes forces are localized and cause concentrated repetitive forces to be transferred from the frame to the diaphragm. These concentrated loads combined with the cyclical nature of the crane loadings (fatigue) make it inadvisable to rely upon diaphragm bracing.

15. WALL SYSTEMS

The special consideration which must be given to wall systems of crane buildings relates to movement and vibration. Columns are commonly tied to the wall system - to provide

bracing to the column or to have the column support the wall. (The latter is applicable only to lightly loaded columns.) For masonry and concrete wall systems it is essential that proper detailing be used to tie the column to the wall. Figure 15.1 illustrates a detail which has worked well for masonry walls.

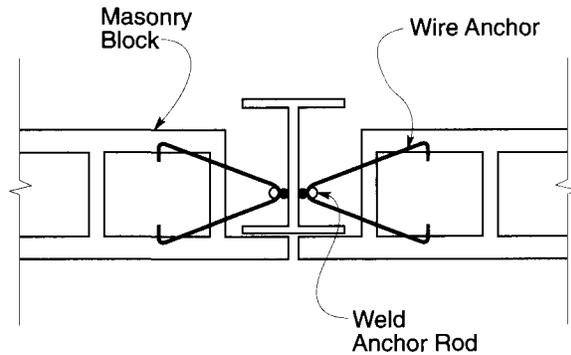


Fig. 15.1 Masonry Wall Anchorage

The bent anchor rod has flexibility to permit movement perpendicular to the wall but are "stiff parallel to the wall, enabling the wall to brace the column about its weak axis. The use of the wall as a lateral bracing system for columns should be avoided if future expansion is anticipated.

If a rigid connection is made between column and wall and crane movements and vibrations are not accounted for, wall distress is inevitable.

16. FRAMING SYSTEMS

The same general comments given previously for industrial buildings without cranes apply to crane buildings as well. However, the most economical framing schemes are normally dictated by the crane. Optimum bays are usually smaller for crane buildings than buildings without cranes and usually fall into the 25 to 30 foot range. This bay size permits the use of rolled shapes as crane runways for lower load crane sizes. 50 to 60 foot main bays, with wind columns, are generally more economical when deep foundations and heavy cranes are specified.

The design of framing in crane buildings must include certain serviceability considerations which are used to control relative and absolute lateral movements of the runways by controlling the frame and bracing stiffness. The source producing lateral movement is either an external lateral load (wind or earthquake) or the lateral load induced by the operation of the crane. The criteria are different for pendant operated versus cab operated cranes since the operator rides with the crane in the latter case. In crane bays with gabled roofs, vertical roof load can cause spreading of the eaves and thus spreading of the crane runways. Conversely, eccentrically bracketed runways on building columns can result in inward tilting of the columns due to the crane loading. This would

cause an inward movement of the runways toward each other. Lastly, the crane tractive force can cause longitudinal movement of the runway either by torsion in the supporting columns where brackets are used or flexing of the frame if rigid frame bents are used for the runway columns. Longitudinal runway movement is rarely a problem where braced frames are used.

Recommended serviceability limits for frames supporting cranes:

1. Pendant operated cranes: Frame drift to be less than runway height over 100, based on 10 year winds or the crane lateral loads on the bare frame. While this limit has produced satisfactory behavior, the range of movements should be presented to the building owner for review because they may be perceived as too large in the completed building.
2. Cab operated cranes: Frame drift to be less than runway height over 240 and less than 2 inches, based on 10 year wind or the crane lateral loads on the bare frame.
3. All top running cranes: Relative inward movement of runways toward each other to be less than a 1/2 inch shortening of the runway to runway dimension. This displacement would be due to crane vertical static load.
4. All top running cranes: Relative outward movement of runways away from each other to be not more than an increase of 1 inch in the dimension between crane runways. The loading inducing this displacement would vary depending on the building location. In areas of roof snow load less than 13 psf, no snow load need be taken for this serviceability check. In areas of roof snow load between 13 psf and 31 psf, fifty percent of the roof snow load should be used. Lastly, in areas of where the snow loads exceed 31 psf, seventy-five percent of the roof snow load should be used.

(The discussion of serviceability limits is also presented in more detail in AISC Steel Design Guide No. 3.)⁽¹⁷⁾

17. BRACING SYSTEMS

17.1 Roof Bracing

Roof bracing is very important in the design of crane buildings. The roof bracing allows the lateral crane forces to be shared by adjacent bents. This sharing of lateral load reduces the column moments in the loaded bents. This is true for all framing schemes (i.e. rigid frames of shapes, plates,

trusses, or braced frames). It should be noted, however, that in the case of rigid frame structures the moments in the frame cannot be reduced to less than the wind induced moments.

Figures 17.1.1, 17.1.2 and 17.1.3 graphically illustrate the concept of using roof bracing to induce sharing of lateral crane loads in the columns. For wind loading all frames and columns are displaced uniformly as shown in Fig. 17.1.1. For a crane building without roof bracing the lateral crane loads are transmitted to one frame line (Fig. 17.1.2) causing significant differential displacement between frames. The addition of roof bracing will create load sharing. Columns adjacent to the loaded frame will share in the load thus reducing differential and overall displacement. (Fig. 17.1.3).

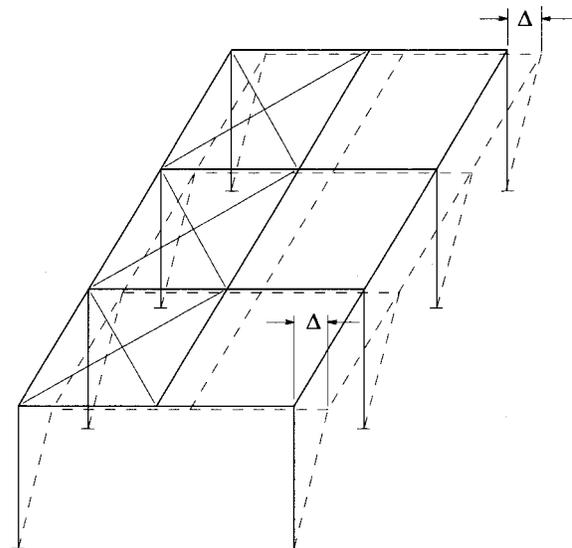


Fig. 17.1.1 Uniform Displacement Due to Wind

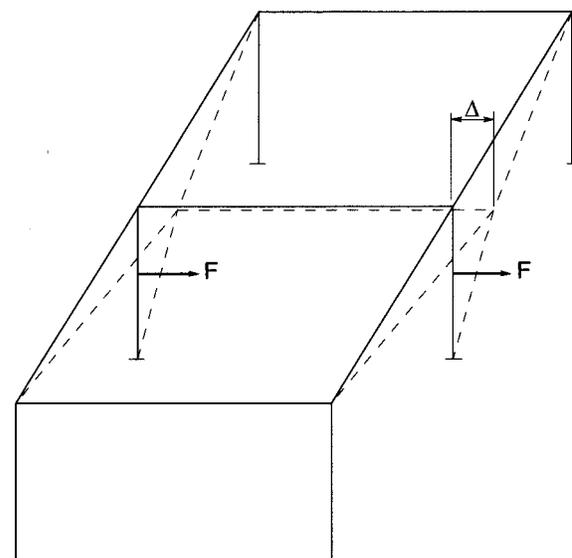


Fig. 17.1.2 Displacement of Unbraced Frames Due to Crane Lateral Load

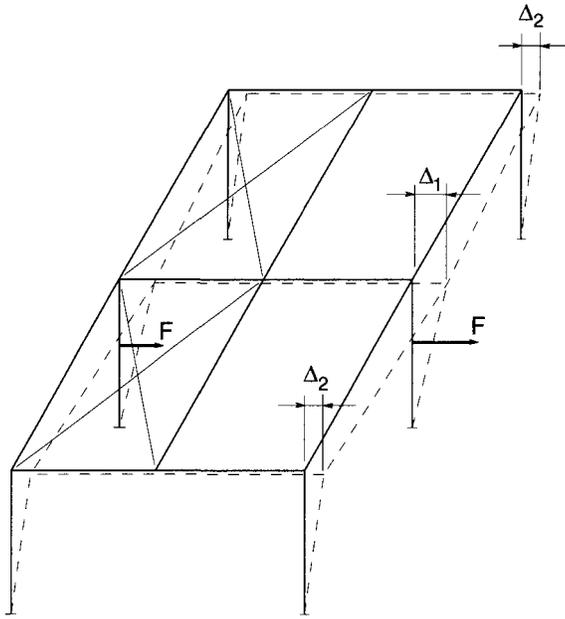


Fig. 17.1.3 Displacement of Braced Frames Due to Crane Lateral Load

Angles or tees will normally provide the required stiffness for this system.

Additional information on load sharing is contained in Section 20.1.

17.2 Wall Bracing

It is important to trace the longitudinal crane forces through the structure in order to insure proper wall and crane bracing (wall bracing for wind and crane bracing may in fact be the same braces).

For lightly loaded cranes, wind bracing in the plane of the wall may be adequate for resisting longitudinal crane forces. (See Fig. 17.2.1) While for very large longitudinal forces, the bracing will most likely be required to be located in the plane of the crane rails. (See Fig. 17.2.2.)

For the bracing arrangement shown in Fig. 17.2.1, the crane longitudinal force line is eccentric to the plane of the X-bracing. The crane column may tend to twist if the horizontal truss is not provided. Such twisting will induce additional stresses in the column. The designer should calculate the stresses due to the effects of the twisting and add these stresses to the column axial and flexural stresses. A torsional analysis can be made to determine the stresses caused by twist, or as a conservative approximation the stresses can be determined by assuming that the twist is resolved into a force couple in the column flanges as shown in Fig. 17.2.3. The bending stresses in the flanges can be calculated from the flange forces. In order to transfer the twist, P_e , into the two flanges, stiffeners may be required at the location of the force P .

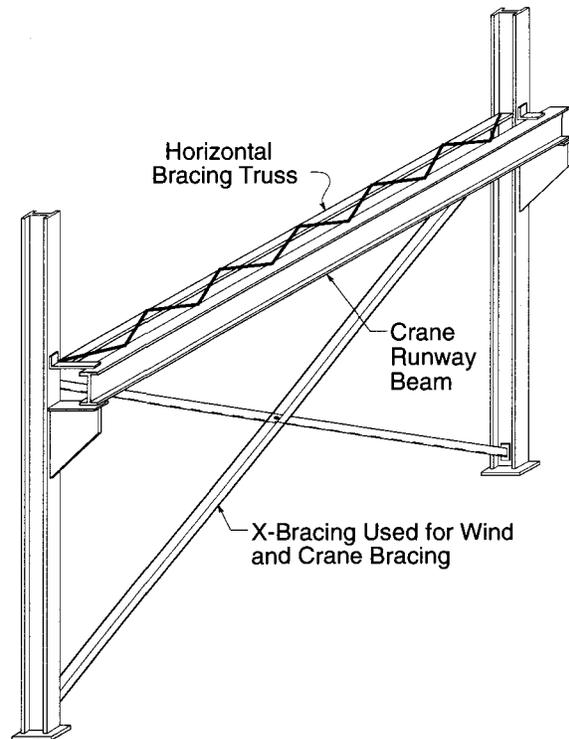


Fig. 17.2.1 Wall Bracing for Cranes

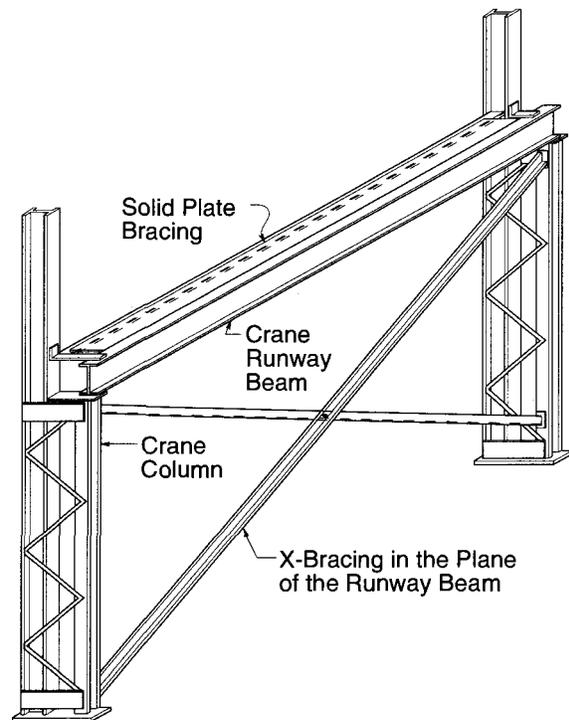


Fig. 17.2.2 Vertical Bracing for Heavy Cranes

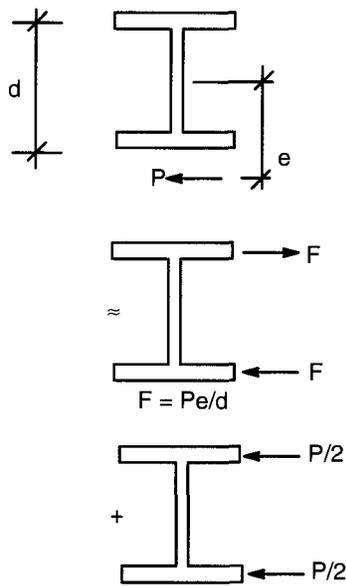


Fig. 17.2.3 Eccentric Column Forces

The following criteria will normally define the bracing requirements for longitudinal crane force transfer:

1. For small longitudinal loads (up to 4 kips) use of wind bracing is generally efficient, where columns are designed for the induced eccentric load.
2. For medium longitudinal loads (4 kips - 8 kips) a horizontal truss is usually required to transfer the force to the plane of X-bracing.
3. For large longitudinal loads (more than 8 kips) bracing in the plane of the longitudinal force is generally the most effective method of bracing. Separate wind X-bracing on braced frames may be required due to eccentricities.

Normally the X-bracing schemes resisting these horizontal crane forces are best provided by angles or tees rather than rods. In cases where aisles must remain open, portal type bracing may be required in lieu of designing the column for weak axis bending. (See Fig. 17.2.4.)

It should be noted that portal bracing will necessitate a special design for the horizontal (girder) member, and that the diagonals will take a large percentage of the vertical crane forces. This system should only be used for lightly loaded, low fatigue situations. The system shown in Fig. 17.2.5 could be used as an alternate to 17.2.4. Shown in Fig. 17.2.6 are possible details for bracing connections at the crane columns. Additional details on connections and bracing can be found in AISC Engineering for Steel Construction⁽¹³⁾.

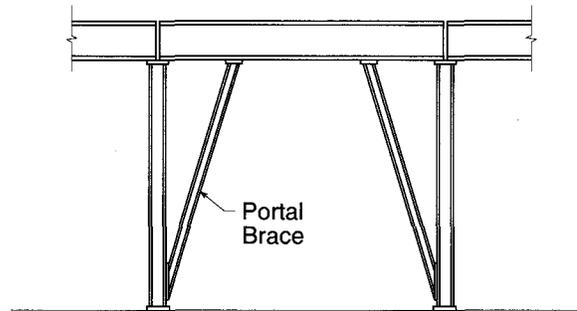


Fig. 17.2.4 Portal Crane Runway Bracing

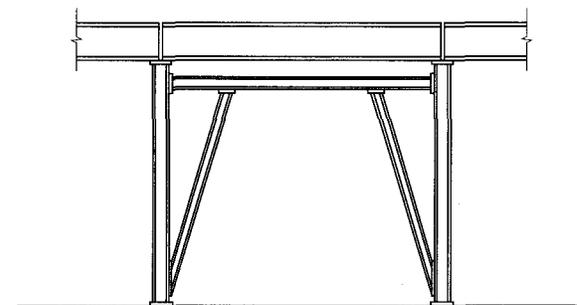


Fig. 17.2.5 Modified Portal Crane Runway Bracing

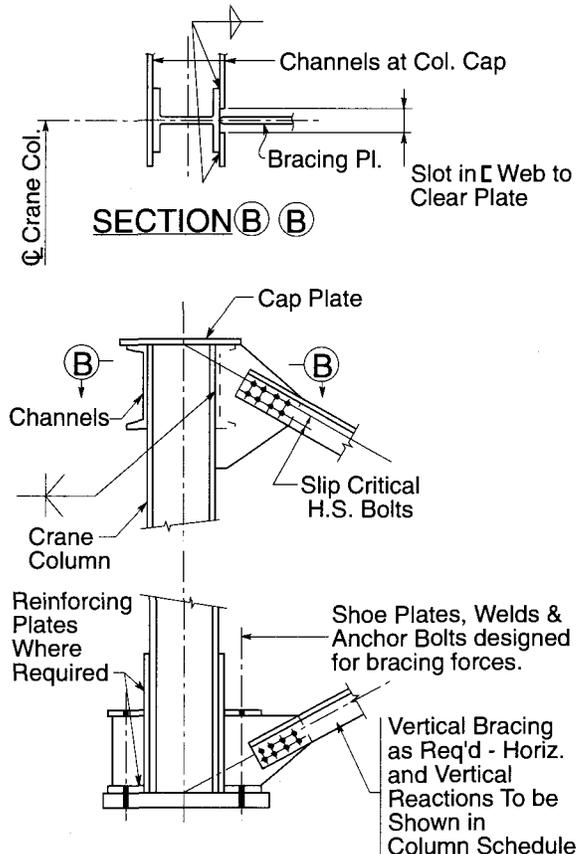


Fig. 17.2.6 Column Brace Details for Fig. 17.2.2

18. CRANE RUNWAY DESIGN

Strength considerations for crane girder design are primarily controlled by fatigue for Class D, E and F cranes. Wheel loads, their spacing, and girder span are required for the design of crane girders. The expense of crane girder construction normally increases when built-up shapes are required. Fatigue restrictions are more severe for built-up shapes (i.e. for built-up sections the difference between a rolled shape vs. a built-up member using continuous fillet welds means a shift from AISC category A to B). This means that for a frequently cycled built-up girder (Loading Condition 4) the allowable stress range is reduced from 24 ksi to 16 ksi.

The following summary of crane girder selection criteria may prove helpful.

1. Light cranes and short spans - use a wide flange beam.
2. Medium cranes and moderate spans use a wide flange beam, and reinforce the top flange with a channel.
3. Heavy cranes and longer spans - use a plate girder, with a horizontal truss or solid plate at the top flange.
4. Limit deflections under crane loads as follows:
Vertical Deflections of the Crane Beam due to wheel loads (no impact):
L/600 Light and Medium Cranes (CMAA Classes A, B, C and D.)
L/1000 Mill Cranes (CMAA Classes E and F.)
Lateral Deflection of the Crane Beam due to crane lateral loads:
L/400 All Cranes.

18.1 Crane Runway Beam Design Procedure

As previously explained, crane runway beams are subjected to both vertical and horizontal forces from the supported crane system. Consequently, crane runway beams must be designed for combined bending about both the X and Y axis.

Salmon⁽³⁵⁾ and Gaylord⁽¹⁹⁾ point out that the equations presented in the AISC Specifications for lateral-torsional buckling strength are based upon the load being applied at the elevation of the neutral axis of the beam. If the load is applied above the neutral axis (for instance, at the top flange of the beam as is the case with crane runway beams), lateral torsional buckling resistance is reduced.. In addition, the lateral loads from the crane system are applied at the top flange level, generating a twisting moment on the beam. When vertical

and lateral loads are applied simultaneously, these two effects are cumulative. To compensate for this, it is common practice to assume the lateral loads due to the twisting moment are resisted by only the top flange. With this assumption, Salmon and Gaylord both suggest that the lateral stability of a beam of this type subject to biaxial bending is otherwise typically not affected by the weak axis bending moment (M_y). Consequently the appropriate allowable bending stress (F_b) for combined bending is based on a yield criterion and is equal to $0.6F_y$ for the unbraced section.

Another criterion related to crane runway beam design is referred to in the AISC Specifications as "sidesway web buckling" (Section K1.5). This criterion is included to prevent buckling in the tension flange of a beam where flanges are not restrained by bracing or stiffeners and are subject to concentrated loads. This failure mode may predominate when the compression flange is braced at closer intervals than the tension flange or when a monosymmetric section is used with the compression flange larger than the tension flange (e.g. wide flange beam with a cap channel). A maximum allowable concentrated load is used as the limiting criteria for this buckling mode.

This criteria does not currently address beams subjected to simultaneously applied multiple wheel loads.

For crane runway beams the following design procedure is recommended as both safe and reasonable where fatigue is not a factor.

1. Compute the required moments of inertia (I_x and I_y) to satisfy deflection control criteria.
L/600 to L/1000 for Vertical Deflection.
L/400 for Lateral Deflection.
2. Position the crane to produce worst loading conditions. This can be accomplished using the equations found in the AISC Manual for cranes with two wheel end trucks on simple spans. For other wheel arrangements the maximum moment can be obtained by locating the wheels so that the center of the span is midway between the resultant of the loads and the nearest wheel to the resultant. The maximum moment will occur at the wheel nearest to the Centerline of the span. For continuous spans the maximum moment determination is a trial and error procedure. Use of a computer for this process is recommended.
3. Calculate Bending Moments (M_x and M_y) including effects of impact.
4. Select a section ignoring lateral load (M_y) effects from:

$$S_x = \frac{M_x}{F_{bx}}$$

where F_{bx} is obtained from AISC Equations in Chapter F.

5. Check this section by using:

$$M_x/S_x + M_y/S_t \leq 0.6 F_y$$

S_t = Section modulus of top half of section about y-axis. For rolled beams without channel caps, S_t should be taken as 1/2 of the total S_y of the shape, since only the top flange resists the lateral crane loads. For sections with channel caps, S_t is the section modulus of the channel and top flange area. Values of this parameter are provided in Appendix A, Table 1, for various W and C combinations. Table 1 also lists values for I_x , S_1 , S_2 and Y_1 . S_1 and S_2 refer to bottom and top flange section moduli respectively, Y_1 is the distance from the bottom flange to the section centroid. Table 1 also gives the moment of inertia of the "top flange" of the combined W and C sections.

6. Check the section with respect to sidesway web buckling as described in Section K1.5 of the AISC Specifications.

In selecting a trial rolled shape section, it may be helpful to recognize that the following ratios exist for various W shapes without channel caps:

| W Shape | S_x/S_y |
|-----------------|-----------|
| W8 through W16 | 3 to 8 |
| W16 through W24 | 5 to 10 |
| W24 through W36 | 7 to 12 |

Table 2 in Appendix A provides the radius of gyration r_x and d/A_f for commonly used channel and wide flange combinations. In addition, for these combinations, the maximum span (unbraced length) for which the allowable bending stress can be taken as $0.6 F_y$ is listed.

Where fatigue is a consideration, the above procedure should be altered so that the live load stress range for the critical case does not exceed fatigue allowables as per AISC Appendix K.

An often overlooked aspect of crane girder design is the local longitudinal bending stresses in the top flange of the runway girder due to the passage of the crane wheel. The fact that such localized stresses exist was first determined from strain gage measurements which showed that top flange stresses were higher than calculated from conventional bending theory.

AISE Technical Report No. 13 provides the following equation for use in calculating this localized stress:

$$f_{bw} = \frac{Pt_f}{8(I_R + I_f)} \left[2(I_R + I_f) \frac{h}{t} \right]^{1/4}$$

where:

- f_{bw} = local longitudinal flange stress due to bending under wheel load, ksi
- P = maximum wheel load, kips
- t_f = thickness of top flange, in.
- I_R = moment of inertia of rail cross-section, in.⁴
- I_f = moment of inertia of top flange, in.⁴
- h = clear depth between flangest, in.
- t = web thickness, in.

This stress is added to the normal bending stress in the flange. Thus, the stress on the top of the top flange will be increased by the value of f_{bw} and the stress on the bottom of the top flange will be decreased by the value of f_{bw} . Experience has shown the value of f_{bw} to be in the range of 1.0 to 4.0 ksi, thus, an increase in top flange compression can be anticipated.

EXAMPLE 18.1.1: Local Wheel Support Stresses.

Assume the following:

1. Maximum wheel load = 65 kips
2. W36x300 crane girder.
3. ASCE 85# rail.

$$f_{bw} = \frac{Pt_f}{8(I_R + I_f)} \left[2(I_R + I_f) \frac{h}{t} \right]^{1/4}$$

$$P = 65 \text{ kips}$$

$$I_f = 1/12(16.655)(1.68)^3 = 6.58 \text{ in.}^4$$

$$I_R = (36.74 - 2 \times 1.68) = 33.38 \text{ in.}^4$$

$$t = 0.945 \text{ in.}$$

$$f_{bw} = \frac{65 \times 1.68}{8(30.1 + 6.58)} \left[2(30.1 + 6.58) \frac{33.38}{.945} \right]^{1/4} = 2.66 \text{ ksi.}$$

EXAMPLE 18.1.2: Crane Runway Girder Design (ASD)

Crane Capacity = 20 Tons (40 kips)

Bridge Span = 70 Ft.

Type of Control - Cab Operated

Bridge Weight = 57.2 kips

Trolley Weight = 10.6 kips

Maximum Wheel Load (without impact) = 38.1 kips

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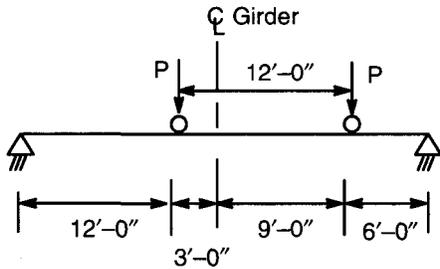
Wheel Spacing = 12'-0"

Runway Girder Span = 30' - 0"

Assume no reduction in allowable stress due to fatigue.

Use AISC criteria and A36 steel.

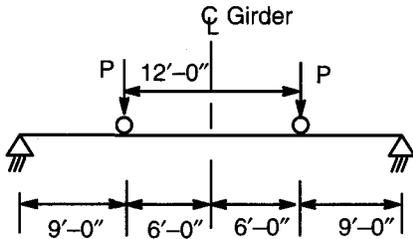
The critical wheel locations with regard to bending moment are:



$$M_{\max.} = \frac{P}{2(30)} \left(30 - \frac{12}{2} \right)^2$$

$$= 9.60(P) \text{ ft. - kips.}$$

The critical wheel locations with regard to deflection are:



$$\Delta_{\max.} = \frac{P(9)}{24(29000)(I)} \left[3(30)^2 - 4(9)^2 \right] (1728)$$

$$= \frac{53.1(P)}{I}$$

At working loads,

Max. Vertical Load/Wheel = 38.1 kips

Max. Horizontal Load/Wheel = $0.20(40+10.6)/4$
 = 2.53 kips (Section 11)

Using the vertical deflection criterion of L/600,

$$(\Delta_x)_{\text{allow.}} = 30(12)/600 = 0.60''$$

$$I_{x-x \text{ req'd.}} = 53.1(38.1)/0.60 = 3372 \text{ in.}^4$$

Using horizontal deflection criterion of L/400,

$$(\Delta_y)_{\text{allow.}} = 30(12)/400 = 0.90''$$

$$I_{y-y \text{ req'd}} \text{ (for top flange)} = 53.1(2.53)/0.90$$

$$= 149 \text{ in.}^4$$

Calculate M_x and M_y :

Assume the girder weight = 125 plf

$$M_x \text{ (including impact)} = 443 \text{ ft-kips}$$

$$= 9.00(38.1)(1.25) + 0.125(30)^2/8$$

$$M_y = 9.00(2.53) = 22.8 \text{ ft-kips.}$$

Assuming $F_{bx} = 22 \text{ ksi}$,

$$(S_1)_{\text{req'd.}} = 443(12)/22 = 242 \text{ in.}^3$$

Using Tables 1 and 2 from Appendix A, try a W27x94 with a C15x33.9 cap channel.

$$I_{x-x} = 4530 \text{ in.}^4 > 3372 \text{ in.}^4 \quad \text{o.k.}$$

$$I_{y-y} \text{ for top flange and cap channel} = 377 \text{ in.}^4 > 149 \text{ in.}^4$$

o.k.

$$S_1 = 267.82 \text{ in.}^3$$

$$S_2 = 435.55 \text{ in.}^3$$

$$S_t = 50.25 \text{ in.}^3$$

Check bending about the x-axis:

$$(f_{bx})_{\text{tension}} = \frac{M_x}{S_1} = \frac{443(12)}{267.82} = 19.8 \text{ ksi}$$

$$(F_{bx})_{\text{tension}} = 0.6F_y = 22 \text{ ksi} > 19.8 \text{ ksi} \quad \text{o.k.}$$

$$(f_{bx})_{\text{compression}} = \frac{M_x}{S_2} = \frac{443(12)}{435.55} = 12.2 \text{ ksi}$$

From Table 2 of Appendix A,

Span = 30' \cong 0.6 F_y span limit therefore:

$$(F_{bx})_{\text{compression}} = 22.0 \text{ ksi} > 12.2 \text{ ksi} \quad \text{o.k.}$$

Check biaxial bending in the top flange.

$$f_{bx} = 12.2 \text{ ksi}$$

$$f_{by} = \frac{22.8(12)}{50.25} = 5.44 \text{ ksi}$$

$$(f_b)_{\text{combined}} = \text{Maximum combined bending stress}$$

$$= 12.2 + 5.44 = 17.64 \text{ ksi}$$

$$(F_b)_{\text{combined}} = 0.6 F_y = 22 \text{ ksi} > 17.64 \text{ ksi} \quad \text{o.k.}$$

Check sidesway web buckling.

Using Equation K1-7 from the AISC ASD Steel Specification,

$$R_{\text{allow.}} = \frac{6800t_w^3}{h} \left[0.4 \left(\frac{d_c / t_w}{L / b_f} \right)^3 \right]$$

$$= \frac{6800(0.49)^3}{25.43} \left[0.4 \left(\frac{24 / 0.49}{360 / 9.99} \right)^3 \right]$$

$$= 31.6 \text{ kips.}$$

Maximum wheel load with impact = 38.1 (1.25)
= 47.6 kips

47.6 > 31.6 kips **N.G.**

Section does not meet sidesway web buckling criteria as described in the AISC ASD Steel Specification. A wide flange, section with a larger flange width would be required. Calculations show a W24x104 w/ MC18x42.7 (A36) or a plane W27x161 (A36) beam to be adequate. See the comments on sidesway web buckling at the end of Example 18.1.3.

Use a W24x104 w/MC18x42.7 (A361 or a W27x161 (A361).

EXAMPLE 18.1.3: Crane Runway Girder Design (LRFD)

Same criteria as used in Example 18.1.2 but design check is per the AISC LRFD Specification.

Calculate factored loads.

The load factors that are currently proposed for crane loads are as follows:

Bridge weight: Load Factor = 1.2.

Trolley weight and lifted load: Load Factor = 1.6.

For the crane used in this example, the factored wheel loads are calculated as follows:

$$P_{\text{factored}} = P_{\text{bridge}} (1.2) + P_{\text{trolley + lifted load}} (1.6)$$

$$P_{\text{bridge}} = 57.2/4 = 14.3 \text{ kips/wheel (38.1 kips included bridge weight)}$$

$$P_{\text{trolley + lifted load}} = 38.1 - 14.3 = 23.8 \text{ kips.}$$

For vertical loads,

$$P_{\text{factored}} = 14.3(1.2) + 23.8(1.6)$$

$$= 55.2 \text{ kips/wheel.}$$

For horizontal loads,

$$P_{\text{factored}} = (10.6 + 40)(1.6)(0.20)/4$$

$$= 4.05 \text{ kips/wheel.}$$

The deflection criteria is based on working loads and therefore is the same as calculated for Example 18.1.2.

Calculate factored M_x and M_y .

Assuming the girder weight to be 125 plf, the factored moments including impact are calculated as:

$$(M_x)_{\text{factored}} = 9.00(55.2)(1.25) + 0.125(30)^2(1.2)/8$$

$$= 638 \text{ ft-kips}$$

$$(M_y)_{\text{factored}} = 9.00(4.05) = 36.5 \text{ ft-kips}$$

Investigate the same W27x94 w/C15x33.9 section reviewed in the ASD solution.

Check bending about the x-axis.

For $L_b \leq L_p$, $M_n = M_p = F_y Z$

For $L_p < L_b \leq L_r$,

$$M_n = C_b \left[M_p - (M_p - M_r) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$

For $L_b > L_r$, $M_n = M_{cr}$

$$L_p = \frac{300 r_y}{F_{yf}}$$

Use r_y of the compression flange as calculated below:

$$(I_{y-y})_{\text{comp. flg.}} = 377 \text{ in.}^4$$

$$A_{\text{comp. flg.}} = 9.99(0.745) + 9.96 = 17.40 \text{ in.}^2$$

$$(r_{y-y})_{\text{comp. flg.}} = \sqrt{\frac{377}{17.40}} = 4.65 \text{ in.}$$

$$\text{Therefore, } L_p = \frac{300(4.65)}{\sqrt{36}} = 232.5 \text{ in.}$$

$$L_b = 30(12) = 360 \text{ in.} > L_p$$

Therefore, $M_n < M_p$.

The calculation of the warping constant (C_w) for a beam with a cap channel is an involved problem. Therefore, rather than solving for L_r directly using Eqn. F1-6, L_r is determined by a trial and error iteration using the following equation for M_{cr} for singly symmetric members as provided in Table A-F1.1 of the LRFD Specification.

$$M_{cr} = \frac{57000 C_b}{L_b} \sqrt{I_y J} \left(B_1 + \sqrt{1 + B_2 + B_1^2} \right) \leq S_{xt} F_y$$

$$B_1 = \left[2.25 [2(I_{yc}/I_y) - 1] \right] \left(\frac{h}{L_b} \right) \sqrt{\frac{I_y}{J}}$$

$$B_2 = 25 \left(1 - \frac{I_{yc}}{I_y} \right) \left(\frac{I_{yc}}{J} \right) \left(\frac{h}{L_b} \right)^2$$

The iterative solution for L_r involves assuming a value for L_b in the above equation, calculating M_{cr} , and then comparing

this calculated value of M_{cr} to M_r as defined in the Specification. This process is iterated until M_{cr} is equal to M_r . The value of L_b that provides this equivalence is equal to L_r .

For this shape, the pertinent geometric properties are as follows:

$$I_y = 439 \text{ in.}^4$$

$$J = 4.03 + 1.02 = 5.05 \text{ in.}^4$$

$$I_{yc} = 377 \text{ in.}^4$$

$$h \cong 27 \text{ in.}$$

$$C_b = 1.0$$

As an initial assumption choose $L_b = 55 \text{ ft.} = 660 \text{ in.}$

$$B_1 = 2.25 \left[2 \left(\frac{377}{439} \right) - 1 \right] \left(\frac{27}{660} \right) \sqrt{\frac{439}{5.05}} = 0.6158$$

$$B_2 = 25 \left(1 - \frac{377}{439} \right) \left(\frac{377}{5.05} \right) \left(\frac{27}{660} \right)^2 = 0.4411$$

Substituting and solving:

$$M_{cr} = 7990 \text{ in.-kips.}$$

Defining S_{xt} as the section modulus about the bottom (tension) flange and S_{xc} as the section modulus about the top (compression) flange.

$$S_{xt} F_y = 267.82(36) = 9462 \text{ in.-kips (tension flange does not control)}$$

$$M_r = (F_y - F_r) S_{xc}$$

Since the channel cap is welded to top flange, use $F_r = 16.5 \text{ ksi.}$

$$M_r = (36 - 16.5)(435.55) = 8493 \text{ in.-kips.}$$

$$7990 < 8493, \text{ therefore } L_b < 55 \text{ ft.}$$

For the second iteration, try $L_b = 53 \text{ ft.}$

For $L_b = 53 \text{ ft. (or } 636 \text{ in.)}$

$$B_1 = 0.6158 (660/636) = 0.6390$$

$$B_2 = 0.4412(660/636)^2 = 0.4751$$

$$M_{cr} = 8488 \text{ in.-kips} \cong \boxed{8493 \text{ in.-kips.}}$$

Therefore, $L_r \cong 53 \text{ ft.} = 636 \text{ in.}$

$$L_p < L_b \leq L_r$$

$$\text{Therefore, } M_n = \left[M_p - (M_p - M_r) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right]$$

$$M_p = F_y Z_x.$$

From table values in the LRFD Manual of Steel Construction, $Z_x = 357 \text{ in.}^3$

$$M_p = 36(357) = 12852 \text{ in.-kips.}$$

$$M_{nx} = \left[12852 - (12852 - 8502) \left(\frac{360 - 232.5}{636 - 232.5} \right) \right]$$

$$= 11477 \text{ in.-kips.}$$

$$\phi M_{nx} = 0.90(11477) = 10329 \text{ in.-kips.}$$

$$M_{ux} = 679(12) = 8148 \text{ in.-kips} < 10329 \text{ in.-kips.}$$

o.k.

Check biaxial bending in the top flange.

As previously discussed, a stability or buckling check on the section for this load case is not required. Since this problem is really a case of combined bending and torsion on the composite section, Equation H2-1 of the Specification can provide the appropriate check for this load case. This equation correlates well with the design check used in the ASD solution.

Equation H2-1 states that:

$$\phi F_y \geq f_{un}$$

$$\phi = 0.90$$

For this case, $f_{un} = (f_{bx} + f_{by})_{\max}$.

$$= \frac{M_{ux}}{S_2} + \frac{M_{uy}}{S_t}$$

$$= \frac{679(12)}{435.55} + \frac{38.9(12)}{50.25}$$

$$= 28.0 \text{ ksi.}$$

$$\phi F_y = 0.90(36) = 32.4 \text{ ksi} > 28.0 \text{ ksi} \quad \text{o.k.}$$

Check sidesway web buckling.

Using Equation K1-7 from the AISC LRFD Steel Specification,

$$R_n = \frac{12000 t_w^3}{h} \left[0.4 \left(\frac{d_c / t_w}{L / b_f} \right)^3 \right]$$

However, according to the Specification, "if the concentrated load is located at a point where the web flexural stress due to factored load is below yielding, 24,000 may be used in lieu of 12,000" in this equation. The previously discussed biaxial bending criteria will ensure that the web flexural stress beneath the load is less than F_y . Therefore, ϕR_n is evaluated as:

$$\phi R_n = \frac{0.85(24000)(.49)^3}{25.43} \left(0.4 \left(\frac{24 / 0.49}{360 / 9.99} \right)^3 \right)$$

$$= 94.8 \text{ kips.}$$

Maximum factored wheel load w/ impact =55.2(1.25)
 =69.0 kips.

69.0 < 94.8 o.k.

W27x94 w/C15x33.9 (A36) is adequate based on LRFD check.

It should be noted that the ASD Specification currently does not have a comparable increase in allowable concentrated load for the sidesway web buckling check when flexural stress in the web is less than $0.6 F_y$. Therefore, there is an inconsistency in the two Specifications (ASD and LRFD) with the ASD Specification providing more conservative criteria. This explains why the W27x94 w/C15x33.9 section was adequate for the LRFD check but inadequate for the ASD check. Although it is generally not recommended that ASD and LRFD design criteria be mixed, since sidesway web buckling is an independent failure mode, it seems reasonable that crane runways designed using ASD procedures can be checked using LRFD equations for sidesway web buckling.

18.2 Plate Girders

Plate girder runways can be designed in the same manner as rolled sections, but the following items become more important to the design.

1. Plate girder runways are normally used in mill buildings where many cycles of load occur. Since they are built-up sections, fatigue considerations are extremely important.
2. Welding stiffeners to the girder webs may produce a fatigue condition which would require reduction in stress range.⁽³¹⁾ Thickening the girder web so that stiffeners are not required (except for the bearing stiffeners which are located at points of low flexural stress) may provide a more economical solution. However, in recent years, numerous cases of fatigue cracks at the junction of the top flange of the girder and the web have been noted. These cracks have been due to:
 - a. The rotation of the top flange when the crane rail was not directly centered over the web. (See Fig. 18.2.1)
 - b. The presence of residual stresses from the welding of the flange and stiffeners to the web.
 - c. Localized stresses under the concentrated wheel loads.

The presence or absence of stiffeners affects problems a. and c. If intermediate stiffeners are eliminated or reduced, the problem of eccentric crane rail location becomes more severe. If intermediate stiffeners are provided, full penetration welds

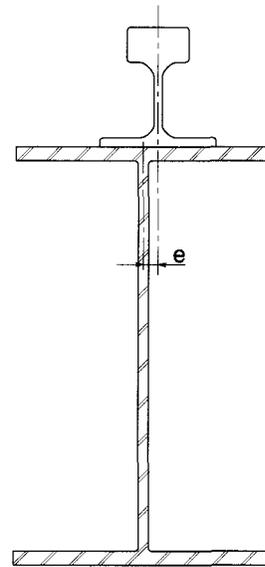
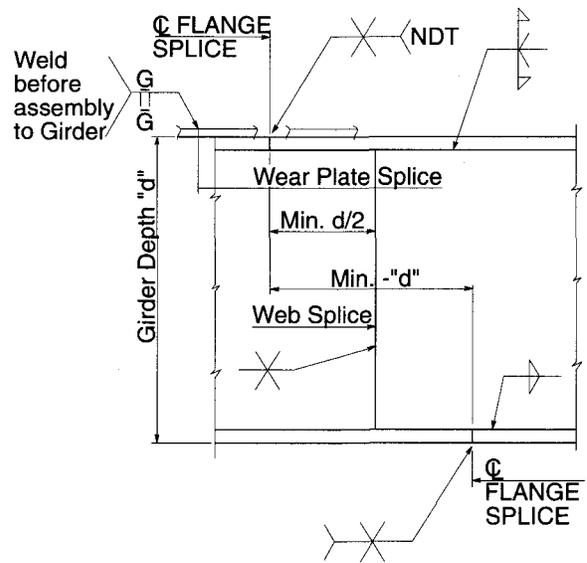


Fig. 18.2.1 Rail Misalignment

should be used to connect the top of the stiffener to the underside of the top flange. At the tension flange the stiffeners should be terminated not closer than 4 times nor more than 6 times the web thickness from the toe of the web-to-flange weld.

Shown in Figs. 18.2.2 through 18.2.7 are details which pertain to heavy crane runway installations. The difference in weld and stiffener detailing between older AISC publications and the stiffeners shown here are generally the result of revised detailing techniques for fatigue conditions.



(Splice mat'l as req'd)
 Grinding to be longitudinal

Fig. 18.2.2 Girder Splice

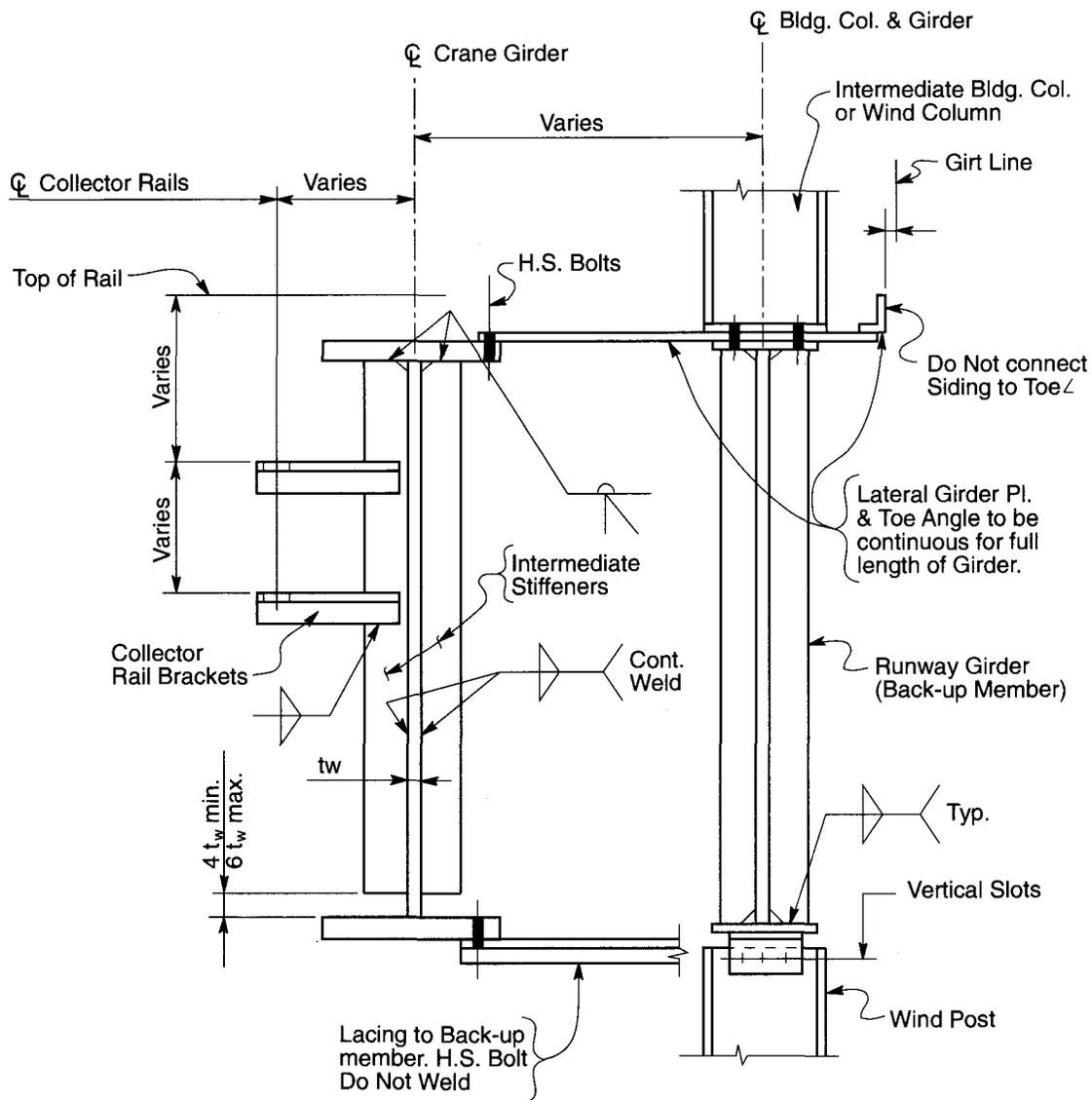
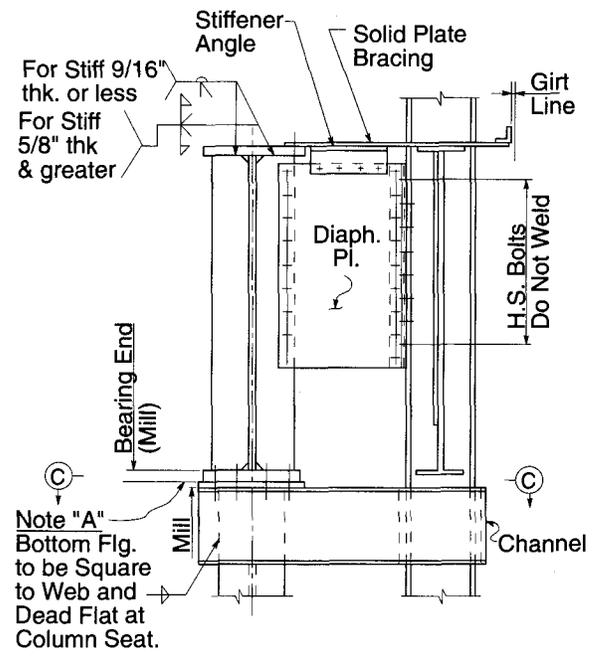
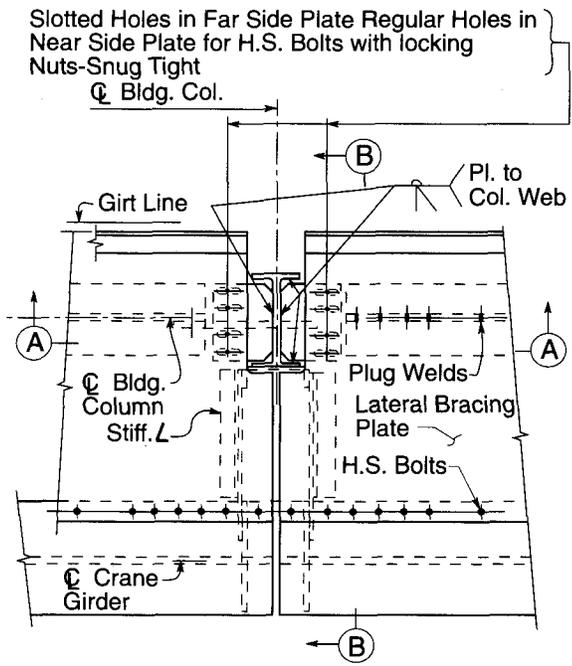


Fig. 18.2.3 Crane Runway Girder Detail



SECTION B B
Crane Gdr. Detail at Bearing Stiffener

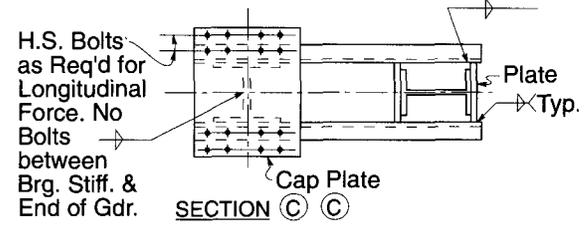
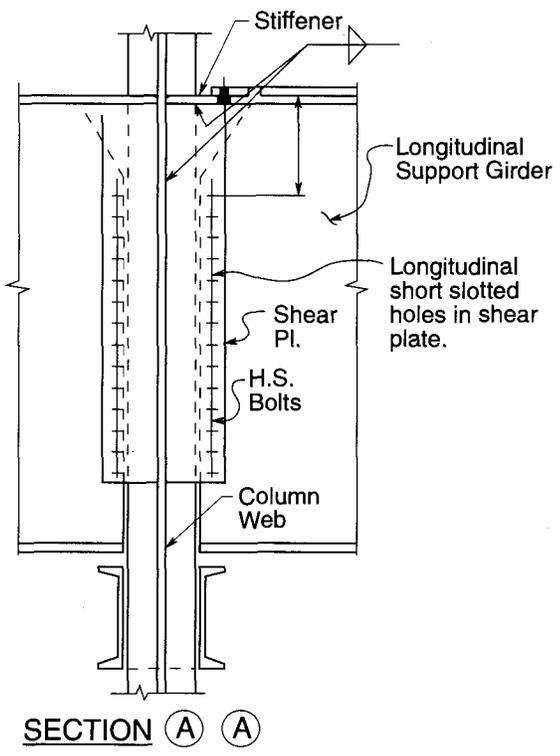


Fig. 18.2.5 Sections A and C of Fig. 18.2.4

Fig. 18.2.4 Detail at Ends of Crane Girders

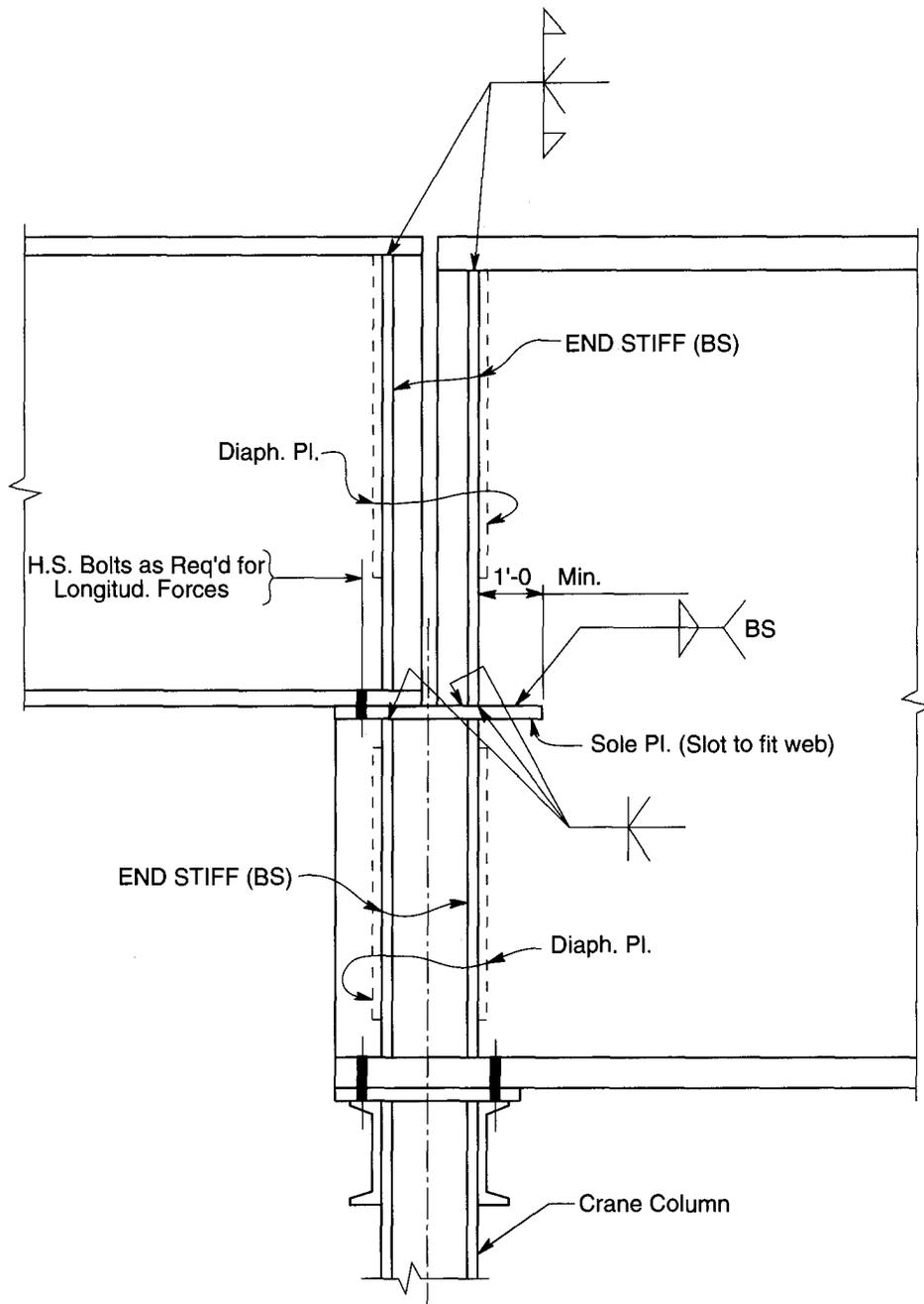


Fig. 18.2.6 Section at Different Depth Crane Girders

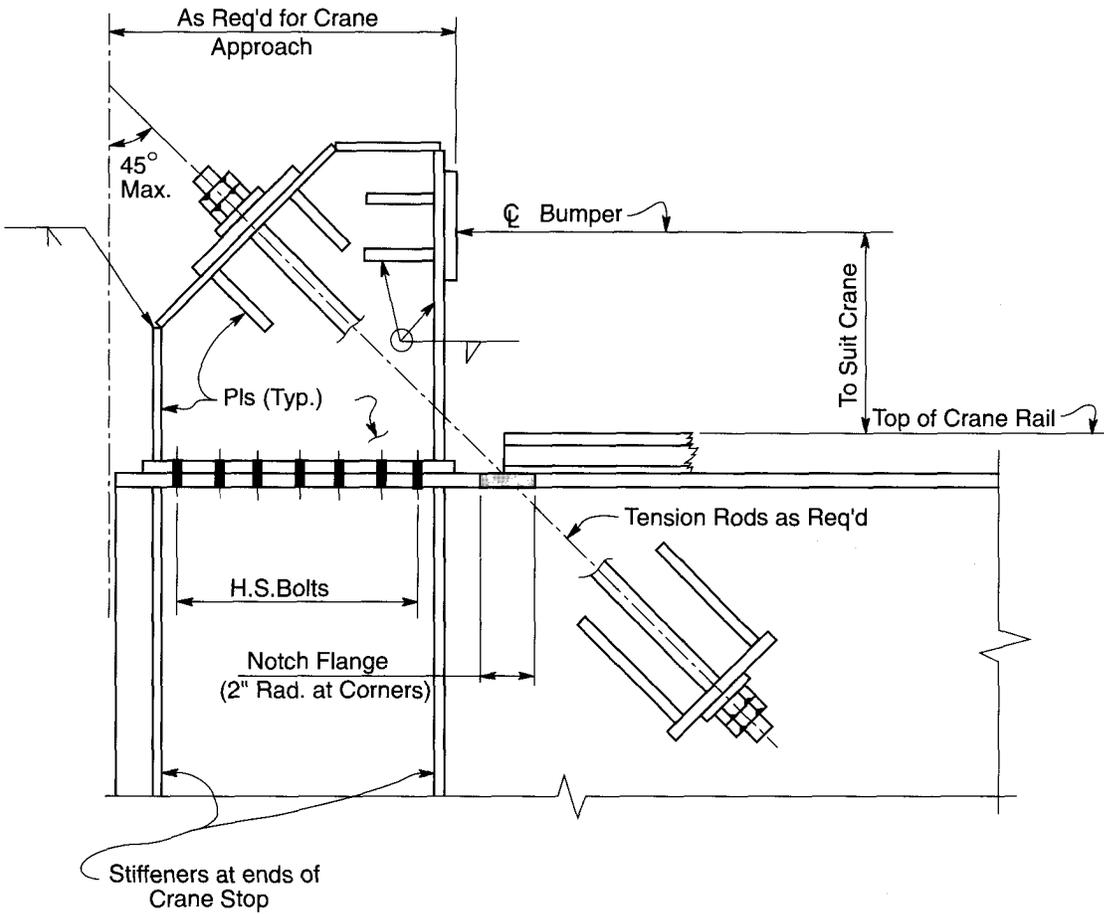


Fig. 18.2.7 Typical Crane Stop

18.3 Simple Span vs. Continuous Runways

The decision as to whether simple span or continuous crane girders should be used has been debated for years. Following is a brief list of advantages of each system. It is clear that each can have an application.

1. Advantages of Simple Span:

- a. Much easier to design for various combinations of loads.

- b. Generally unaffected by differential settlement of the supports.
- c. More easily replaced if damaged.
- d. More easily reinforced if the crane capacity is increased.

2. Advantages of Continuous Girders:

- a. Continuity reduces deflections which quite often control.
- b. End rotations and movements are reduced.

- c. Result in lighter weight shapes and a savings in steel cost when fatigue considerations are not a determining factor.

Continuous girders should not be used if differential settlement of the supports is of the magnitude that could cause damage to the continuous members. Also, when continuous girders are subjected to fatigue loading and have welded attachments on the top flange (rail clips) the allowable stress range is reduced considerably. Any advantage therefore may be eliminated.

Shown in Fig. 18.3.1 are the results of several runway designs for spans from 20 to 30 feet. A36 and 50 ksi steel designs were made for a 4 wheel, 10T crane, with a 70' bridge for continuous (two span) vs. simple span conditions. In these examples, deflection did not control. Fatigue was not considered. The curves represent (in general) the trends for heavier cranes as well. In general, the use of two span continuous crane girders could save about 18% in weight over simply supported girders.

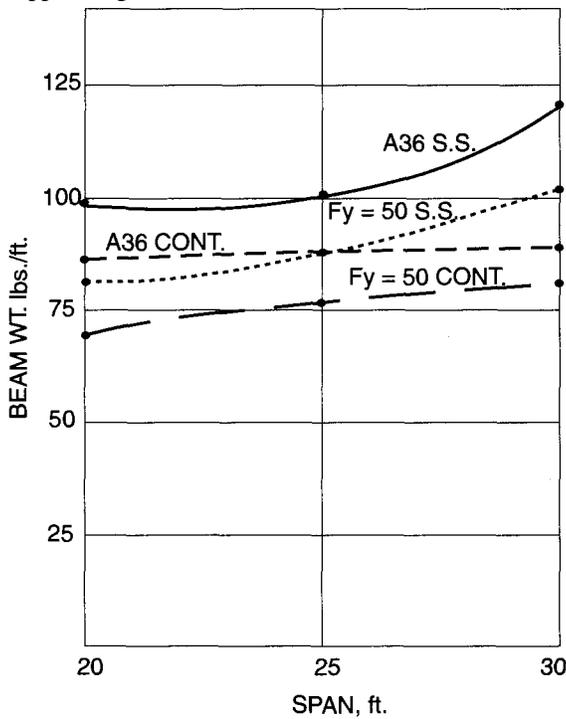


Fig. 18.3.1 Weights of Runways

18.4 Channel Caps

Use of channel caps is normally required to control lateral deflections and to control the stresses due to lateral loads. For light duty-lightweight cranes (less than 5T) channel caps may not be required. Studies have found that a steel savings of approximately 15 lbs/ft, is required to justify the cost of welding a cap to a structural shape.

18.5 Runway Bracing Concepts

An excellent paper on the subject of bracing of crane girders is that of Mueller.⁽²⁶⁾ Several significant (and common) considerations that need to be emphasized are:

1. As illustrated in Figure 2 in the Mueller paper (repeated here as Fig. 18.5.1), improper detailing at the end bearing condition could lead to a web tear in the end of the crane girder. The detail shown in Figure 18.5.2 has been used to eliminate this problem for light crane systems. The details shown in Fig. 18.2.3 and 18.2.4 would represent a similar detail for heavy cranes. Use of this detail allows the end rotation and yet properly transfers the required lateral forces into the column.

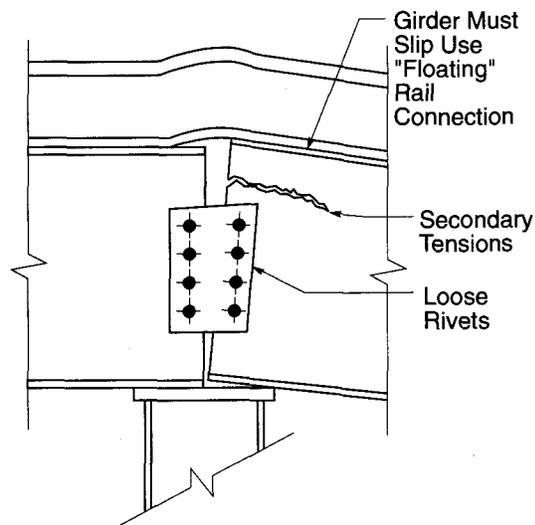


Fig. 18.5.1 Improper Girder Connection Detail

2. A common method of bracing the crane girder is to provide a horizontal truss (lacing) or a horizontal plate to connect the crane girder top flange to an adjacent structural member as previously illustrated in Figures 17.2.1 and 17.2.2.

A critical consideration in the use of this system is to have the lacing flexible in the vertical direction, enabling the crane girder to freely deflect relative to the structural member to which it is attached. If the lacing is not flexible, stresses will be produced which could cause a fatigue failure of the lacing system, thereby losing the lateral support for the girder.

3. AISE Technical Report No. 13 requires that girders more than 36 feet in length must have the bottom flange braced by a horizontal truss system. Where compliance with this AISE Standard is not required many engineers have used a bottom flange channel to brace the flange on long spans (perhaps 40 feet or

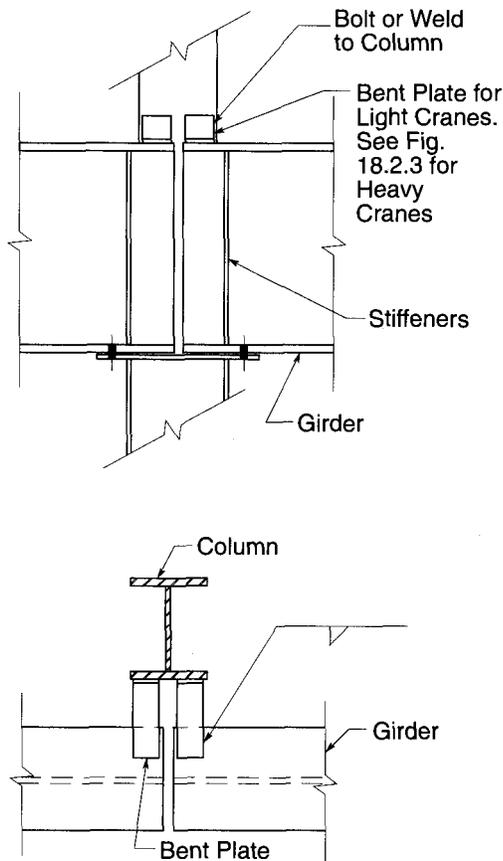


Fig. 18.5.2 Proper Tie Back Detail

more). The origin of this requirement is not obvious; however, it appears that compliance with the AISC sidesway web buckling equations may analytically satisfy this requirement.

4. Occasionally two parallel crane girders are connected by a top plate to "mutually brace each other". This, of course, results in a very stiff girder in terms of lateral load. Also, the plate can be used as a walkway for maintenance purposes. When tied together the loading and unloading of parallel girders can cause a fatigue failure of the bracing plate unless it is properly detailed. The interconnecting plate must be flexible to allow differential deflections between two girders. Also if the horizontal plate is used it is likely that OSHA⁽³⁰⁾ will consider the plate a footwalk, thus OSHA requirements must be met.

18.6 Crane Stops

The end section of a crane runway must be designed for a longitudinal force applied to the crane stops. For spring type bumper blocks the longitudinal crane stop force may be calculated from the following formula.

$$\text{where } F = \frac{WV^2}{gc_t}$$

- W = total weight of crane exclusive of lifted load.
- V = specified crane velocity at moment of impact, fps (required by AISE Technical Report No. 6 to be 50% of full load rated speed.
- c_t = stroke of spring at point where the crane stopping energy is fully absorbed, ft.
- F = total longitudinal inertia force acting at the elevation of the center of mass of the bridge and the trolley. The force on each runway stop is the maximum bumper reaction from the inertia force acting at such locations,
- g = acceleration of gravity, 32.2 fps²

For bumper blocks of wood or rubber (commonly found in older cranes) the above equation is not directly applicable. Manufacturers literature or experience must be used for such installations. In the absence of specific data, it is recommended that the designer assume the bumper force to be the greater of:

1. Twice the tractive force, or
2. Ten percent of the entire crane weight.

18.7 Crane Rail Attachments

There are four general types of anchoring devices used to attach crane rails to crane runway beams. These types are hook bolts, rail clips, rail clamps and patented clips. Details of hook bolts and rail clamps are shown in the AISC manual.

18.7.1 Hook Bolts

Hook bolts provide an adequate means of attachment for light rails (40 lbs. - 60 lbs.) and light duty cranes (CMAA Classes A, Band C). The advantage of hook bolts are: 1) they are relatively inexpensive, 2) there is no need to provide holes in the runway beam flange and 3) it is easy to install and align the rail. They are not recommended for use with heavy duty cycle cranes (CMAA Classes D, E and F) or with heavy cranes (greater than 20 ton lifting capacity), because hook bolts are known to loosen and/or elongate. It is generally recommended that hook bolts should not be used in runway systems which are longer than 500 feet because the bolts do not allow for longitudinal movement of the rail. Because hook bolts are known to loosen in certain applications, a program of periodic inspection and tightening should be instituted for runway systems using hook bolts. Designers of hook bolt attachments should be aware that some manufacturers supply hook bolts of smaller than specified diameter by the use of up-set threads.

18.7.2 Rail Clips

Rail clips are forged or cast devices which are shaped to match specific rail profiles. They are usually bolted to the runway girder flange with one bolt or are sometimes welded. Rail clips have been used satisfactorily with all classes of cranes. However, one drawback is that when a single bolt is used the clip can rotate in response to rail longitudinal movement. This clip rotation can cause a camming action thus forcing the rail out of alignment. Because of this limitation rail clips should only be used in crane systems subject to infrequent use, and for runway systems less than 500 feet in length.

18.7.3 Rail Clamps

Rail clamps are a common method of attachment for heavy duty cycle cranes. Rail clamps are detailed to provide two types: tight and floating. Each clamp consists of two plates: an upper clamp plate and a lower filler plate.

The lower plate is flat and roughly matches the height of the toe of the rail flange. The upper plate covers the lower plate and extends over the top of the lower rail flange. In the tight clamp the upper plate is detailed to fit tight to the lower rail flange top, thus "clamping" it tightly in place when the fasteners are tightened. In the past, the tight clamp had been illustrated with the filler plates fitted tightly against the rail flange toe. This tight fit-up was rarely achieved in practice and is not considered to be necessary to achieve a tight type clamp. In the floating type clamp, the pieces are detailed to provide a clearance both alongside the rail flange toe and below the upper plate. The floating type does not in reality clamp the rail but merely holds the rail within the limits of the clamp clearances. High strength bolts are recommended for both clamp types.

Tight clamps are generally preferred and recommended by crane manufacturers because they feel that the transverse rail movement allowed in the floating type causes accelerated wear on crane wheels and bearings.

Floating rail clamps may be required by crane runway and building designers to allow for longitudinal movement of the rail thus preventing (or at least reducing) thermal forces in the rail and runway system.

Because tight clamps prevent longitudinal rail movement, they should not be used in runways greater than 500 feet in length. Since floating rail clamps are frequently needed and crane manufacturers' concerns about transverse movement are valid, a modified floating clamp is required. In such a clamp it is necessary to detail the lower plate to a closer tolerance with respect to the rail flange toe. The gap between lower plate edge and flange toe can vary between snug and a gap of 1/8". The 1/8" clearance allows a maximum of 1/4" float for the system. This should not be objectionable to crane

manufacturers since this amount of float is within normal CMAA tolerances for crane spans in the range of 50-100 feet, i.e. spans usually encountered in general construction. In order to provide field adjustment for variations in the rail width, runway beam alignment, beam sweep and runway bolt hole location, the lower plate can be punched with its holes off center so that the plate can be flipped to provide the best fit. An alternative would be to use short slotted or oversize holes. In this case one must rely on bolt tightening to clamp the connection so as to prevent filler plate movement.

Rail clamps are generally provided with two bolts per clamp. Two bolts are desirable in that they prevent the camming action described in the section on forged or cast rail clips. A two bolted design is especially recommended if clamps of the longitudinal expansion type described above are used. Rails are sometimes installed with pads between the rail and the runway beam. When this is done the lateral float of the rail should not exceed 1/32", to reduce the possibility of the pads being worked out from under the rail.

18.7.4 Patented Rail Clips

This fourth type of anchoring device covers various patented devices for crane rail attachment. Each manufacturer's literature presents in detail the desirable aspects of the various designs. In general they are easier to install due to their greater range of adjustment while providing the proper limitations of lateral movement and allowance for longitudinal movement. Patented rail clips should be considered as a viable alternative to conventional hook bolts, clips or clamps. Because of their desirable characteristics patented rail clips can be used without restriction except as limited by the specific manufacturer's recommendations. Installations using patented rail clips sometimes incorporate pads beneath the rail. When this is done the lateral float of the rail should be limited as in the case of rail clamps.

18.7.5 Design Of Rail Attachments

The design of rail attachments is largely empirical. The selection of the size and spacing of attachments is related to rail size. This relation is reasonable in that rail size is related to load.

With regard to spacing and arrangement of the attachment the following recommendations are given. Hooked bolts should be installed in opposing pairs with three to four inches between the bolts. The hook bolt pairs should not be spaced further than two feet apart. Rail clips and clamps should be installed in opposing pairs. They should be spaced three feet apart or less.

In addition to crane rail attachment, other attachments in the form of clips, brackets, stiffeners, etc. are often attached to the crane girder. These attachments are often added by plant

engineering personnel. Welding should only be done after a careful engineering evaluation of its effects. Welding (including tack welding) can significantly shorten the fatigue life. Therefore:

1. Never weld crane rail to girder.
2. Clamp, screw or bolt all attachments to crane girders to avoid fatigue problems.
3. All modifications and repair work must be submitted to engineering for review and approval before work is done.

18.8 Crane Rails and Crane Rail Joints

The selection of rail relates to crane considerations (basically crane weight) and is generally made by the crane manufacturer. Once this decision is made, the principal consideration is how the rail sections are to be joined. There are several methods to join rails but two predominate at the present time.

The bolted butt joint is the most commonly used rail joint. Butt joint alignment is created with bolted splice plates. These plates must be properly maintained (bolts kept tight). If splice bars become loose and misaligned joints occur, a number of potentially serious problems can result, including chipping of the rail, bolt fatigue, damage to crane wheels, and as a result of impact loading, increased stresses in the girders. Girder web failures have been observed as a consequence of this problem.

The welded butt joint, when properly fabricated to produce full strength, provides an excellent and potentially maintenance free joint. However, if repairs are necessary to the rails, the repair procedure and consequently the down time of plant operations is generally longer than if bolted splices had been used. The metallurgy of the rails must be checked to assure use of proper welding techniques, but if this is accomplished the advantages can be significant. Principal among these is the elimination of joint impact stresses, existent in non welded rail construction, resulting in reduced crane wheel bearing wear.

Rail joints should be staggered so that the joints do not line up on opposite sides of the runway. The amount of stagger should not equal the spacing of the crane wheels and in no case should the stagger be less than one foot.

Rail misalignment is the single most critical aspect of the development of high impact and lateral stresses in crane girders. Proper use and maintenance of rail attachments is critical in this regard. Rail attachments must be completely adjustable and yet be capable of holding the rail in alignment. Because the rails may become misaligned regular maintenance is essential to correct the problem.

One aspect of crane rail design is the use of crane rail pads. These are generally preformed fabric pads that work best with welded rail joints. The resilient pads will reduce fatigue, vibration and noise problems. Reductions in concentrated compression stresses in the web have been achieved with the use of these pads. Significant reductions in wear to the top of the girder flange have also been noted. With the exception of a few patented systems, the pads are generally not compatible with floating rail installations since they can work their way out from under the rail. Also prior to using a pad system careful consideration to the cost benefits of the system should be evaluated.

19. CRANE RUNWAY FABRICATION & ERECTION TOLERANCES

Crane runway fabrication and erection tolerances should be addressed in the project specifications because standard tolerances used in steel frameworks for buildings are not tight enough for buildings with cranes. Also, some of the required tolerances are not addressed in standard specifications.

Tolerances for structural shapes and plates are given in the Standard Mill Practice section of the Manual of Steel Construction published by AISC. These tolerances cover the permissible variations in geometrical properties and are taken from ASTM Specifications, AISI Steel Product Manuals and Producer's Catalogs. In addition to these Standards, the following should be applied to crane runways.

- a. Sweep: not to exceed 1/4 inch in a 50 foot beam length.
- b. Camber: not to vary from the camber given on the drawing by plus or minus 1/4 inch in a 50 foot beam length.
- c. Squareness: within 18 inches of each girder end the flange shall be free of curvature and normal to the girder web.

Columns, base plates and foundations should adhere to the following tolerances.

- a. Column anchor bolts shall not deviate from their theoretical location by 0.4 times the difference between bolt diameter and hole diameter through which the bolt passes.
- b. Column base plates: Individual column base plates shall be within $\pm 1/16$ inch of theoretical elevation and be level within ± 0.01 inches across the plate length or width. Paired base plates serving as a base for double columns shall be at the same level and not vary in height from one to another by 1/16 inch.

Crane runway girders and crane rails shall be fabricated and erected for the following tolerances.

- a. Crane rails shall be centered on the Centerline of the runway girders. The maximum eccentricity of center of rail to Centerline of girder shall be three-quarters of the girder web thickness.
- b. Crane rails and runway girders shall be installed to maintain the following tolerances.
 - 1. The horizontal distance between crane rails shall not exceed the theoretical dimension by $\pm 1/4$ inch measured at 68° F.
 - 2. The longitudinal horizontal misalignment from straight of rails shall not exceed $\pm 1/4$ inch in 50

feet with a maximum of $\pm 1/2$ inch total deviation in the length of the runway.

- 3. The vertical longitudinal misalignment of crane rails from straight shall not exceed $\pm 1/4$ inch in 50 measured at the column centerlines with a maximum of $\pm 1/2$ inch total deviation in the length of the runway.

The foregoing tolerances were taken from AISE Technical Report No. 13. The following Table shown in Fig. 19.1 is taken from MBMA's Low Rise Building Systems Manual and gives an alternative set of runway erection tolerances.

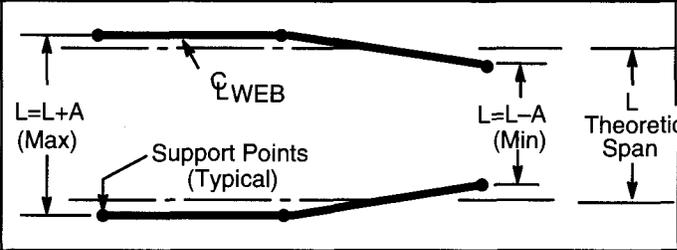
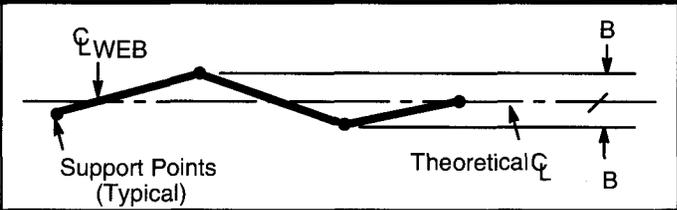
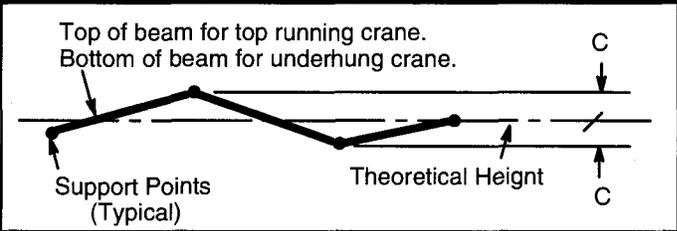
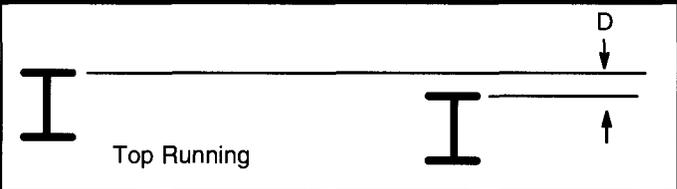
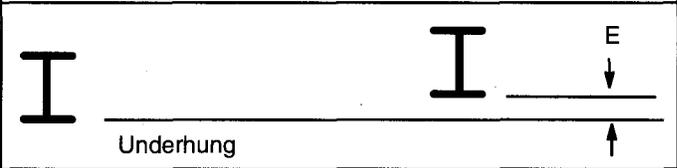
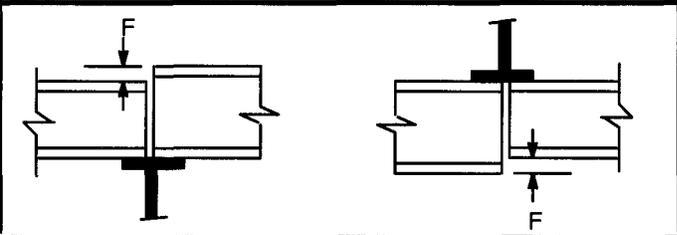
| Item | | Tolerance | Maximum Rate of Change |
|-----------------------------|---|-----------|------------------------|
| Span |  | A=3/8" | 1/4"/20' |
| Straightness |  | B=3/8" | 1/4"/20' |
| Elevation | <p>Top of beam for top running crane. Bottom of beam for underhung crane.</p>  | C=3/8" | 1/4"/20' |
| Beam to Beam Top Running |  | D=3/8" | 1/4"/20' |
| Beam to Beam Underhung |  | E=3/8" | 1/4"/20' |
| Adjacent Beams |  | F=1/8" | N/A |

Fig. 19.1 MBMA Crane Runway Erection Tolerances

20. COLUMN DESIGN

No attempt will be made to give a complete treatise on the design of steel columns. The reader is referred to a number of excellent texts on this subject.⁽¹⁹⁾⁽³⁵⁾

This section of the guide includes a discussion of the manner in which a crane column can be analyzed, how the detailing and construction of the building will affect the loads the crane column receives, and how shears and moments will be distributed along its length. The guide also includes a detailed example of a crane column to illustrate certain aspects of the design.

In most crane buildings, the crane columns are structurally indeterminate. Normally the column is "restrained" at the bottom by some degree of base fixity. The degree of restraint is to a large extent under the control of a designer, who may require either a fixed base or a pinned base.

It is essential to understand that the proper design of crane columns can only be achieved when column moments are realistically determined. This determination requires a complete frame analysis in order to obtain reliable results. Even if a complete computer frame analysis is employed, certain assumptions must still be made of the degree of restraint at the bottom of a column and the distribution of lateral loads in the structure. Further, in many cases a preliminary design of these crane columns must be performed either to obtain approximate sizes for input into a computer analysis or for preliminary cost and related feasibility studies. Simplifying assumptions are essential to accomplish these objectives.

20.1 Base Fixity and Load Sharing

Crane columns are constructed as bracketed, stepped, laced, or battened columns. (See Figure 20.1.1.) In each case,

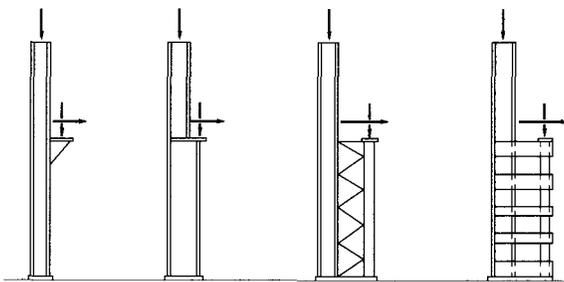


Fig. 20.1.1 Column Types

the eccentric crane loads and lateral loads produce moments in the columns. The distribution of column moments is one principal consideration.

For a given loading condition, the moments in a crane column are dependent on many parameters. Most parameters (e.g. geometry, nonprismatic conditions) are readily accom-

modated in the design process using standard procedures. However, two parameters which have a marked effect on columns moments are:

1. Base fixity.
2. Amount of load sharing with adjacent bents.

As an example, refer to Figure 20.1.2. The loading consists of 100T crane (vertical crane load = 310 kips, lateral crane load to each side = 23 kips). A stepped column is used, but the same general principles apply to the other column types.

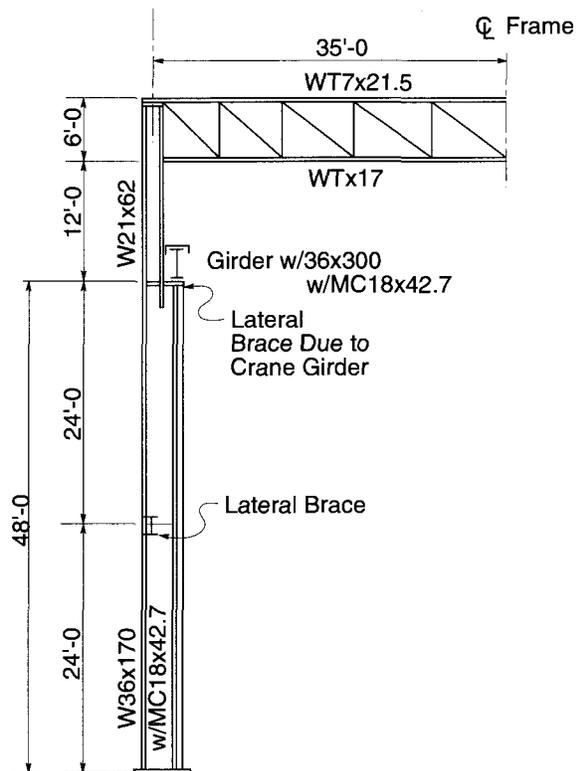


Fig. 20.1.2 Example Frame

1. **Base Fixity:** The effect of base fixity on column moments was determined by a computer analysis for the frame for fixed and pinned base conditions. The results of the analysis shown in Figure 20.1.3 demonstrate that a simple base will result in extremely large moments in the upper portion of the column and the structure will be much more flexible as compared to a fixed base column. For fixed base columns the largest moment is carried to the base section of the column where it can, in the case of the stepped column, be more easily carried by the larger section.

It is frequently argued that taking advantage of full fixity cannot be achieved in any practical detail. However, the crane induced lateral loads on the crane column are of short duration, and for such short term loading an "essentially fixed" condition can normally be achieved through proper de-

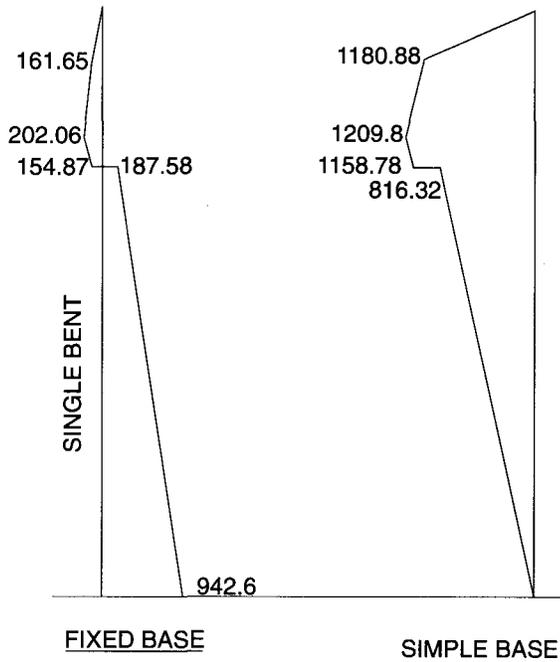


Fig. 20.1.3 Analysis Results

sign. The reduced column moments (22 percent in the previous example) due to the fixed base condition provide good economy without sacrificing stiffness.

There will be cases where subsoil conditions, existing construction restrictions, property line limitations, etc., will preclude the development of base fixity and the hinged base must be used in the analysis. Although the fixed base concept as stated is deemed appropriate due to short term nature of crane loadings, for other long duration building loads the assumption of full fixity may be inappropriate. The reader is referred to an excellent article by Galambos⁽¹⁸⁾ which deals with the effects of base fixity on the buckling strength of frames.

2. Load Sharing To Adjacent Bents: If a stiff system of bracing is used (i.e., a horizontal bracing truss as shown in Figure 20.1.4) then the lateral crane forces and shears can be distributed to adjacent bents thereby reducing column moments. Note that such bracing does not reduce column moments induced by wind, seismic or roof loads but only the singular effects of crane loads. Figure 20.1.5 depicts the moment diagram in the column from a frame analysis based on lateral crane loads being shared by the two adjacent frames (i.e. two-thirds of the lateral sway force is distributed to other frames). The significant reductions in moment are obvious when compared to Figure 20.1.3. (Note the "two-thirds" is an arbitrary distribution used at this point only to illustrate the concept and the significant advantage to be gained. The following paragraphs describe in detail how load sharing actually occurs and how it can be evaluated.

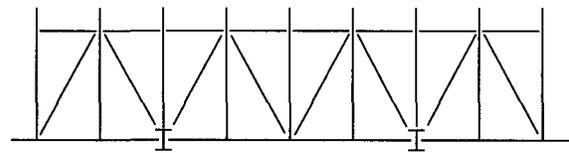


Fig. 20.1.4 Horizontal Bracing

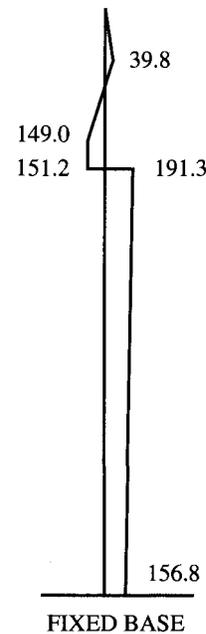


Fig. 20.1.5 Moment Diagram with Load Sharing

Consider a portion of a roof system consisting of five frames braced as shown in Figure 20.1.6. The lateral crane force will result in a reactive force at the level of the lower chord of the roof truss. (Figure 20.1.7.) The distribution of this reactive force to the adjacent frames can be obtained by stiffness methods. This is accomplished by analyzing the horizontal bracing system as a truss on a series of elastic supports. The supports are provided by the building frames and have linear elastic spring constants equal to the reciprocal of the displacement of individual frames due to a unit lateral load (Figure 20.1.8). The model is depicted in Figure 20.1.9. The springs are imaginary members which provide the same deflection resistance as the frames.

This procedure has been programmed and analyzed for many typical buildings. It is obvious that the degree of load sharing varies, and is dependent upon the relative stiffness of the bracing to the frames; however, it was found that for usual horizontal bracing systems a lateral load applied to a single interior frame will be shared almost equally by at least 5 frames. This is logical because bracing of reasonable proportions

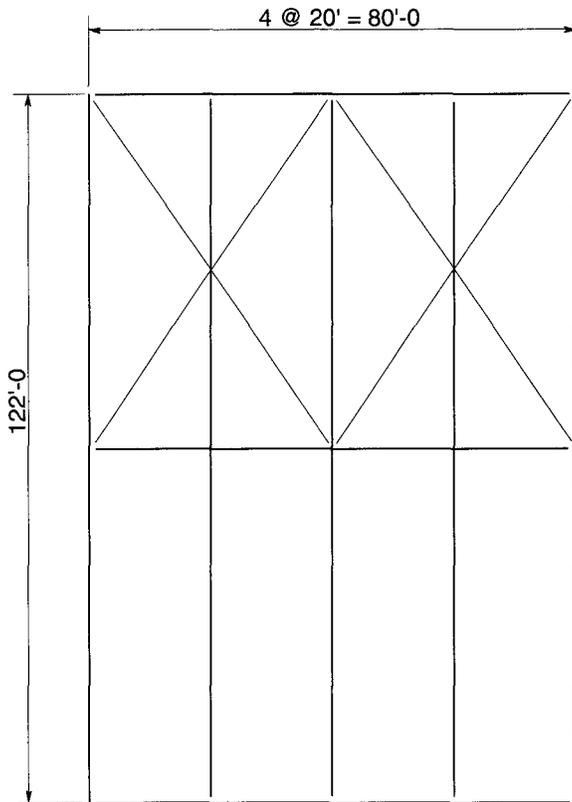


Fig. 20.1.6 Roof Portion

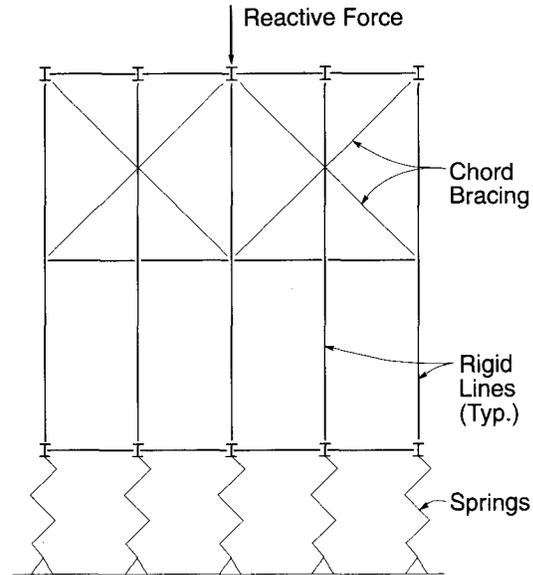


Fig. 20.1.9 Computer Model

made up of axially loaded members is many times as stiff as the moment frames which are dependent upon the bending stiffness of their components.

A building supporting a 100 ton crane is used to illustrate the effect of load sharing. A roof system consisting of five frames X-braced as shown in Figure 20.1.6 was analyzed to determine the force in each frame due to a 20 kip force applied to the center frame. This 20 kips represents the reactive force at the elevation of the bottom chord bracing due to a horizontal crane thrust at the top of the crane girder as illustrated in Figure 20.1.7. The final distribution is shown in Figure 20.1.10.

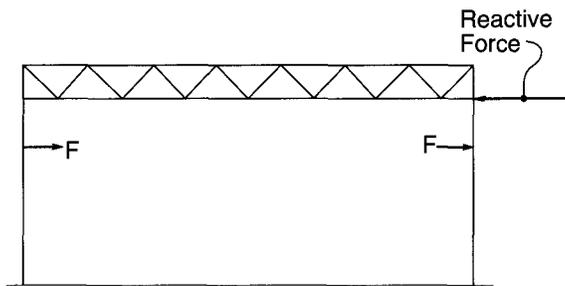


Fig. 20.1.7 Reactive Force

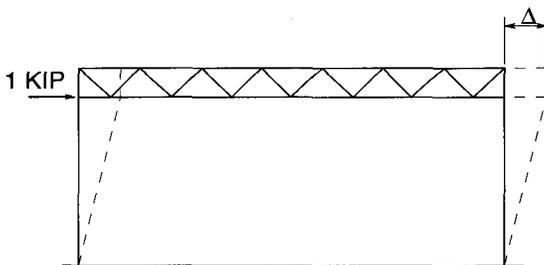


Fig. 20.1.8 Unit Lateral Load

Even though reasonable truss type bracing will distribute a concentrated lateral force to at least 5 frames, it is recommended that load sharing be limited to 3 frames (the loaded frame plus the frame to either side). The reason for this conservative recommendation is that unless pretensioned the horizontal bracing truss members may tend to sag even though "draw" is provided. Thus, a certain amount of movement may occur before the truss "takes up" and becomes fully effective in distributing the load to adjacent frames.

The designer may conclude that if load sharing occurs that a simple method to handle the analysis is to design a given column for one-third the lateral load. This is wrong and unsafe! Each individual crane column must be designed for the full lateral force of the crane. It is only the reactive force applied at the level of the bracing which is distributed to the adjacent frames. *The results of this analysis must be added to or compared with the results of other analyses which are unaffected by the load sharing, i.e., gravity, wind, and seismic loadings.*

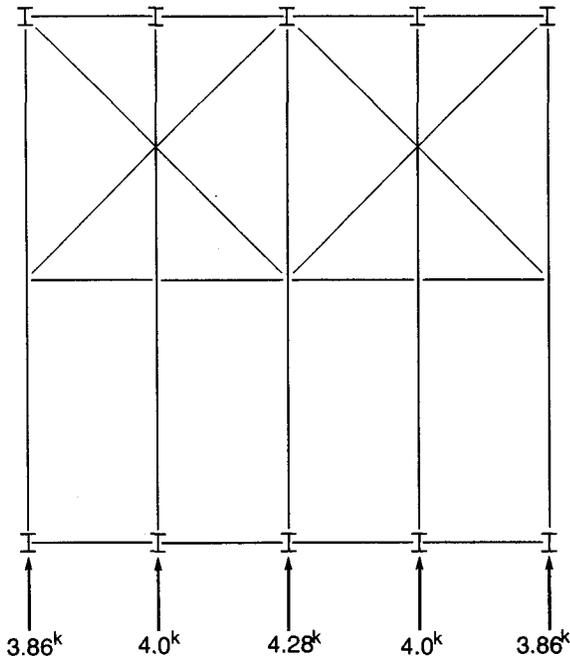


Fig. 20.1.10 Final Force Distribution

To summarize, the most economical designs will result when the following "assumptions" are designed into the structure:

1. Fixed base columns.
2. Horizontal bracing truss (unless wind loads control) such that lateral crane loads can be distributed to adjacent columns.
3. When the roof frames are fabricated trusses the most economical bracing truss location is at the elevation of the bottom chord where they are generally easier to erect. The bottom chord bracing system that is required for uplift and slenderness ratio control may also be adequate for distributing concentrated lateral forces.

20.2 Preliminary Design Methods

Preliminary design procedures for crane columns are especially helpful due to the complexity of design of these members. Even with the widespread availability of computers a good preliminary design can result in substantial gains in overall efficiency. The preceding sections of this guide have pointed out the fact that in order to obtain meaningful column moments a frame analysis is required. A reliable hand calculation method for preliminary design is not only helpful but essential in order to reduce final design calculation time.

The frame analysis which is required to obtain an "exact" solution accomplishes the following:

1. It accounts for sidesway.
2. It properly handles the restraint at the top and at the base of the column.
3. It accounts for non-prismatic member geometry.

What is needed for a preliminary design procedure is a method of analysis that will provide suitable column stiffness estimates so that an "exact" indeterminate frame analysis procedure need be conducted only once. The model given below in Figure 20.2.1 has been found to give remarkably good results for crane loadings, providing horizontal bracing is used in the final design. It is a "no-sway" model, consisting of a fixed base, and supports introduced at the two points where the truss chords intersect the column.



Fig. 20.2.1 No-Sway Computer Model

A moment diagram obtained from the "no-sway" model for the 100 ton crane column previously shown in Fig. 20.1.2 is shown in Figure 20.2.2.

Comparing Figure 20.2.2 to Figure 20.1.5 it can be seen that the general moment configuration is similar, and the magnitudes of moments are almost identical. For preliminary design purposes the two-support "no-sway" model is adequately accurate. The two support no-sway model is statically indeterminate to the second degree. Thus, even a preliminary design requires a complex analysis and certain other assumptions.

The preliminary design procedure for wind or seismic loadings can usually be made by assuming an inflection point

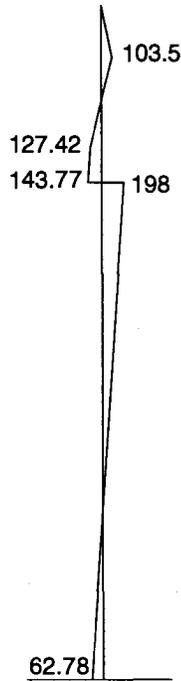


Fig. 20.2.2 Results of No-Sway Model

and selecting preliminary column size to control sway under wind loads. An appropriate procedure is shown in the bracketed crane column design example in the next section.

The sizes of bracketed columns are often controlled by wind; therefore, the design should first be made for wind and subsequently checked for wind plus crane. AISE recommends that bracket vertical loads should be limited to 50 kips.

Stepped and laced or batted columns are another matter. To obtain accurate values for moments, the effects of the nonuniform column properties must be included in the analysis. In doing a preliminary analysis of a stepped column another assumption is practical. The assumption involves the substitution of a single top hinge support to replace the two supports in the two support no-sway mode. The single hinged support is located at the intersection of the bottom chord and the column.

The simplified structure is depicted in Figure 20.2.3. Equations for the analysis of this member are given in Figure 20.2.4.

In each case, the equation for the top shear force is given. For the single support assumption the indeterminacy is eliminated once this shear force is known. The moment diagram for the single hinge, no-sway column evaluated using the equations is given in Figure 20.2.5.

While the variation in moment along the length is not in good agreement with that of the "exact" solution given in Fig-

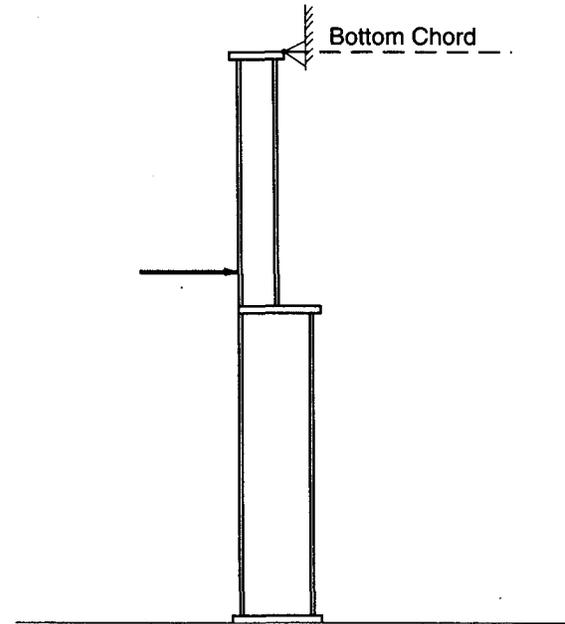


Fig. 20.2.3 Simplified Structure

ure 20.1.5, the values and signs of the moments at critical sections agree quite well.

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There is one aspect of preliminary design that has not been discussed which is essential in the handling of the stepped and double column conditions. The non-prismatic nature of these column types requires input of the moment of inertia of the upper and lower segments of the column which, of course, are not known initially. Therefore, some guidelines and/or methods are required to obtain reasonable values for I_1 and I_2 .

20.2.1 Obtaining Trial Moments of Inertia for Stepped Columns:

The upper segment of the stepped column may be sized by choosing a column section based on the axial load acting on the upper column portion. Use the appropriate unsupported length of the column in its weak direction and determine a suitable column from the column tables contained in the AISC manual. Select a column about three sizes (by weight) larger to account for the bending in the upper shaft.

The size of the lower segment of the stepped column may be obtained by assuming that the gravity load from the crane is a concentric load applied to one flange (or flange-channel combination). The preliminary selection may be made by choosing a member such that $P/A \cong 0.45F_y$ where A is the area of one flange or flange plus channel combination. The depth of the lower shaft is normally determined by the crane clearance requirements (see Figure 20.2.6).

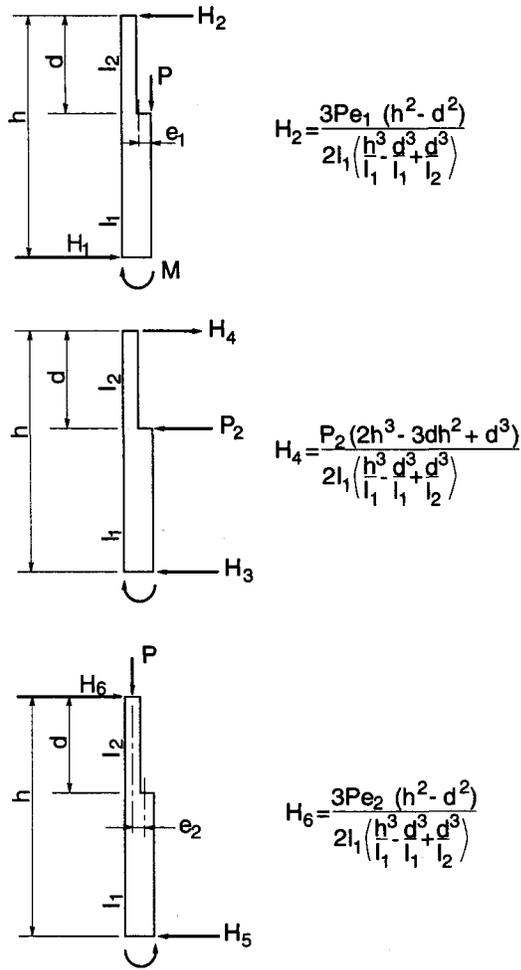


Fig. 20.2.4 Equations for Simplified Structure

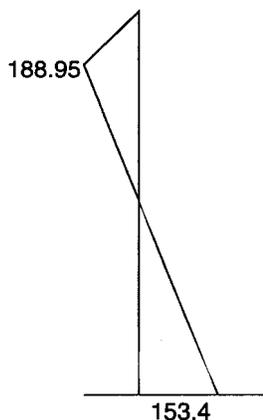


Fig. 20.2.5 Column Moments Using Fig. 20.2.4 Equations

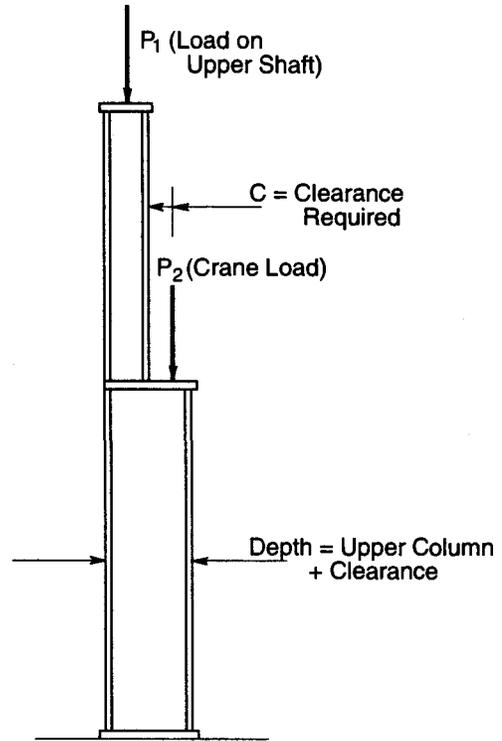


Fig. 20.2.6 Column Clearance Requirement

20.2.2 Obtaining Trial Moments of Inertia for Double Columns:

The building column portion of a double column can again best be selected based on the applied axial load. Select the size of the crane column based on the crane gravity load applied to the "separate" crane column. The allowable stress of this portion will normally be based on the major axis of the column assuming that the column is laced or batted to the building column to provide support about the weak axis. The actual sizes of the columns should be increased slightly to account for the bending moments. The moment of inertia of the combined sections can be calculated using standard formulas for geometrical properties of built-up cross sections. If the moment of inertia of the combined sections is obtained by assuming composite behavior, the lacing or batten plates connecting the two column sections must be designed and detailed accordingly.

20.3 Final Design Procedures

After obtaining the final forces and moments in the crane column, it can be designed. The design of a crane column is unique in that the column has both a varying axial load and a "concentrated" moment at the location of the bracket or "step" in the column.

The best design approach for prismatic bracketed columns is to design the upper and lower portions of the columns

as individual segments with the top portion designed for P_1 and the associated upper column moments, and the lower portion designed for $P_1 + P_2$, and the lower column moments (Figure 20.3.1). The column can normally be considered to be

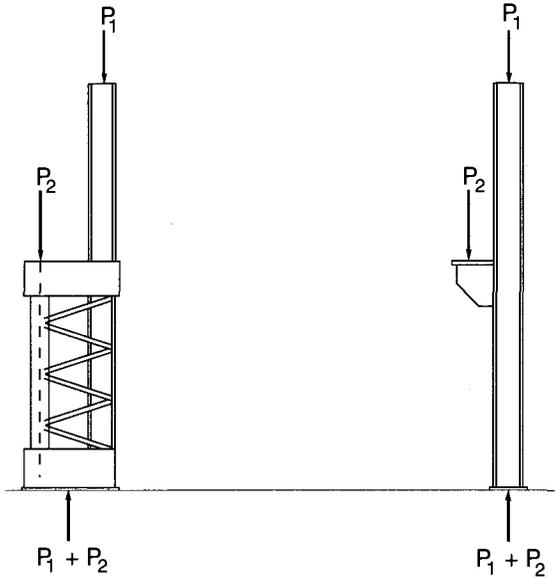


Fig. 20.3.1 Column Loads

laterally braced about the y-axis at the crane girder elevation. When considering the x-axis, F_a , F'_{ex} and K should be calculated based on the entire length of the column and the properties of the cross section. C_m can be assumed to be 0.85 since each column segment is free to sway. A formal theoretical treatment of this procedure can be found in The Design of Steel Beam-Columns, by Peter F. Adams, published by the Canadian Steel Industries Construction Council.⁽¹⁾ The best reference for the design of crane columns is contained in the AISE Technical Report No. 13, Specifications for the Design and Construction of Mill Buildings. The AISE procedure suggests that two equations be checked.

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left[1 - \left(f_a / F'_{ex}\right)\right] F_{bx}} + \frac{C_{my} f_{by}}{\left[1 - \left(f_a / F'_{ey}\right)\right] F_{by}} \leq 1.0$$

$$\frac{f_a}{.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

These equations are nearly identical with equations H1-1 and H1-2 of the ASD AISC Specification for members subjected to both axial compression and bending stresses, except for the introduction of f'_a as different from f_a . In addition, the other terms are in some cases evaluated in a manner adapted especially to the stepped-column problem.

The terms in these equations are defined as follows:

- f_a - In the lower shaft $f_a = (P_1 + P_2)/A$ where A is the area of the lower shaft. In the upper shaft, $f_a = P_1/A$, with A the area of the upper (building) shaft.
- f'_a - In checking the lower shaft for bending about the y-y axis, it is conservatively assumed that the crane support segment resists all of the bending introduced by eccentricity of the crane girder reactions. The amplifications of f_{by} as a result of deflection is dependent on the average axial stress (f'_a) in the crane segment alone. The stress f'_a is determined by adding (or subtracting) the average stress due to moment about the x-axis, calculated at the centroid of the crane segment, to (or from) the average stress f_a of the entire lower shaft.
- F_a - The allowable axial stress under axial load. It may be determined for buckling of the entire stepped column about the x-x axis, based on the equivalent length KL/r_x , or by buckling about the y-y axis for whatever column length is unsupported, in either the upper or lower shaft. It is to be taken as the minimum of the two values in each of the two sets pertinent to the upper and lower shafts, respectively. Exterior wall girts are often assumed not to provide longitudinal (lateral) support to the columns in mill buildings because building alterations may result in their removal. If support in the x direction is provided only at locations A, B and C (Figure 20.3.2) the equivalent length KL for buckling about the y-y axis should be taken as the full unsupported length AB in checking the upper shaft. In checking the lower shaft for the y-y axis, the equivalent length KL should be taken as 0.8 of length BC if the base is assumed to be fully fixed, or as length BC if base fixity against rotation cannot be assured.
- C_{mx} - For bending about the x-x axis, use a value of 0.85 when all bents are under simultaneous wind load and sidesway is assumed to take place. When one bent is being considered, under maximum crane loading, without wind (AISE Case 2 loading) assume a value of 0.95 for C_{mx} .
- C_{my} - Since the crane segment of the lower shaft is assumed to resist all of the bending about the y-y axis, this term is applied to the lower shaft (f_{by} is assumed zero in the upper shaft). Assuming fixity at the base but no interaction with the building column, half of the moment introduced at B as a result of unequal reactions from adjacent girders will be carried down to the base, in

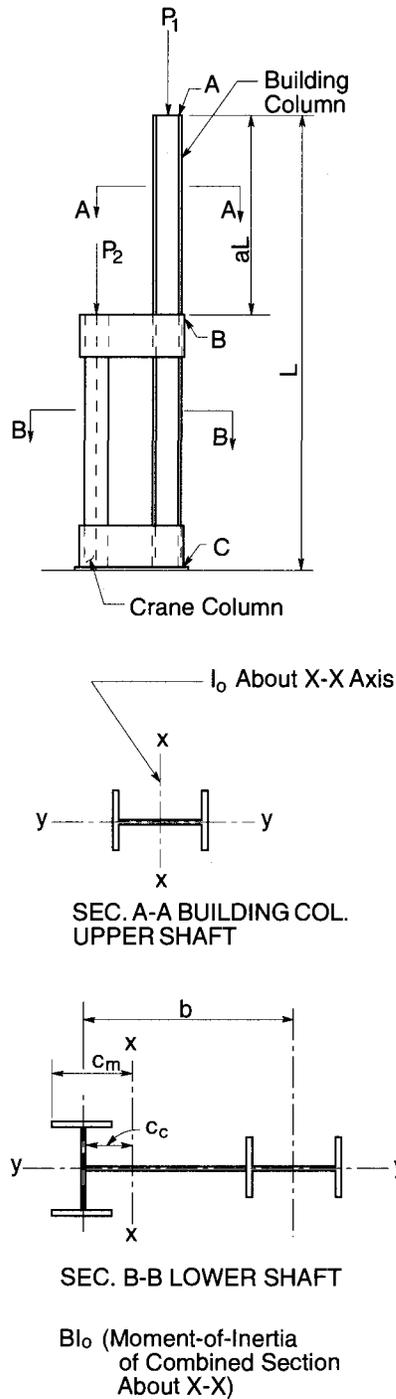


Fig. 20.3.2 Typical Column

which $C_{my} = 0.4$ (AISC Specification Section H1). If base fixity cannot be assumed, take $C_{my} = 0.6$ (hinged condition at base), or, in intermediate situations, interpolate between 0.4 and 0.6.

f_{bx} - Maximum stress due to bending about the x-x axis, assuming an integral action of crane and

building column segments in the lower shaft, and the building column alone in the upper shaft.

f_{by} - Maximum stress due to bending about the y-y axis in the crane column segment of the lower shaft; usually zero in upper shaft.

F_{bx} - For compression on the crane column side of the lower shaft, F_{bx} is the permissible extreme fiber stress due to bending about the x-x axis, reduced if necessary below $0.6F_y$ because of lack of lateral support. The reduced allowable stress may be based on the permissible axial stress in the crane column segment for buckling about the y-y axis as shown in Figure 20.3.2. (The y-y axis in this sketch would correspond to the x-x axis of the individual wide flange segment in the AISC Manual.) The permissible column stress, so determined, should be multiplied by the ratio C_m/C_c , as defined by Section B-B in Figure 20.3.2. In no case is the allowable stress to be greater than $0.6F_y$.

F_{by} - Since this component of bending is about the weak axis of the combined crane and building columns, no reduction in permissible stress need be made for lateral buckling. Also, since the bending resistance is assumed to be provided solely by the crane segment of the lower shaft, the allowable stress for a compact section may be used if the provisions of Section F2 of the AISC Specification are met.

F'_{ex} - Since this stress is used as a basis for the determination of the amplification of column deflection in the plane of bending, it should be based on the equivalent length of the completed stepped column, as in the case of F_a , for bending about the x-x axis.

F'_{ey} - If the base may be assumed as fixed let $K = 0.8$ for the crane column segment alone; otherwise assume $K = 1.0$. The length in the determination of KL is that of the column segment BC.

Example 20.3.1 will illustrate the procedure.

Contained in the AISE publication are effective length values for stepped columns, in terms of three parameters: the ratio of the length of the reduced section to the total length of the column; B , the ratio of the maximum moment-of-inertia of the combined column cross section to that of the reduced section; and P_1/P_2 , the ratio of the axial force in the upper segment to the crane force in the lower segment. (See Figure 20.3.2.)

The AISE tables do not address column end conditions other than fixed or hinged and often times the ratios of P_1/P_2

fall outside the scope of the tables. Contained in Reference 2 and reproduced in Appendix B are tables which address seven different column end conditions, these include:

- a) Pinned - Pinned
- b) Fixed - Free
- c) Fixed - Pinned
- d) Fixed - Slider
- e) Fixed - Fixed
- f) Pinned - Fixed
- g) Pinned - Slider

In addition, these tables include prismatic and nonprismatic columns, and virtually all combinations of P_1 and P_2 load ratios.

EXAMPLE 20.3.1: Bracketed Crane Column Design

Design the column shown in Fig. 20.3.3:

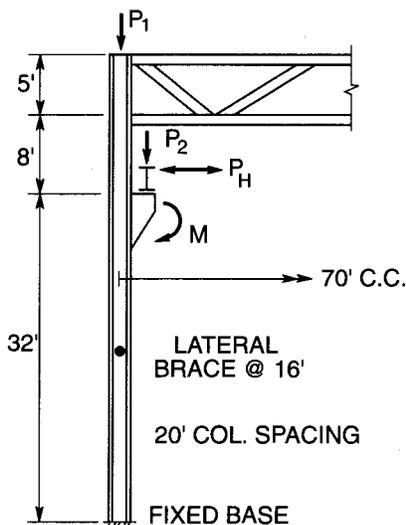


Fig. 20.3.3 Example

Use AISE provisions and A36 steel.

1. Load Cases:

A frame analysis was performed using the AISE load combinations. The critical moment diagrams were obtained from Load Cases 2 and 3.

Case 2 = DL+LL+Crane (Lateral and Vertical)

Case 3 = (DL+Crane Vertical + Wind) .75

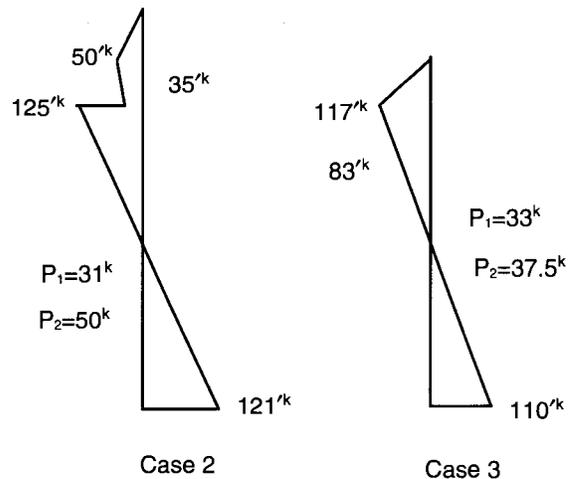


Fig. 20.3.4 Critical Moment Diagrams

Case 2 produces the most critical moment condition in the lower portion of the column, and Case 3 in the upper portion.

2. Preliminary Design:

Since this structure is quite high it is very likely that lateral sway movement could control the column size. Thus, it is recommended that the preliminary design of the column be based on deflection considerations.

Base the allowable sway on:

$$\frac{H}{240} = \frac{45 \times 12}{240} = 2.25'' \quad \text{Use } 2.0''$$

For a fixed-fixed column:

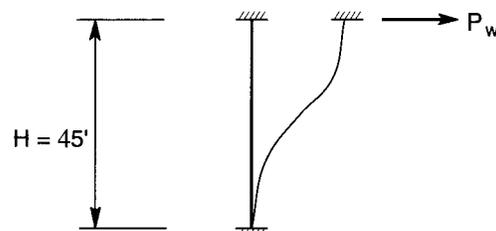


Fig. 20.3.5 Sway Calculation

$$\Delta = \frac{P_w H^3}{24EI} \quad \text{See Fig. 20.3.5.}$$

$$P_w = (WL)(\text{BAY SPACING})(H/2)$$

$$P_w = 20 \times 20 \times 45/2 = 9.0^k$$

Assuming P_w is divided equally between both columns.

$$I_x = \frac{9.0 \times 45^3 \times 1728}{24 \times 30000 \times 2.0} = 984 \text{ in.}^4$$

Try a W16x77.

$$I_x = 1110 \text{ in.}^4$$

3. Stress Check:

The properties of the W16x77 are:

$$\begin{aligned} &= 1110 \text{ in.}^4, & r_x &= 7.0 \text{ in.} & r_T &= 2.77 \text{ in.} \\ &= 134 \text{ in.}^3, & r_y &= 2.47 \text{ in.} \\ &= 22.6 \text{ in.}^2 & d/A_f &= 2.11 \end{aligned}$$

Lower Column Check:

From Case 2, $P_1=31$ kips, $P_2=50$ kips

Use the effective length charts contained in the Appendix B to determine K_x . Assume the column base is fixed and the column top is a fixed roller.

$$\begin{aligned} I_1/I_2 &= 1.0 \\ L_2/L_T &= 32/42.5 = 0.75 \\ P_2/P_T &= 50/81 = 0.62 \end{aligned}$$

Note that L_T is based on the midheight of the roof truss.

By interpolation from the tables: $K_2=0.97$

$$\begin{aligned} KL_x &= (0.97)(42.5) = 41.23 \text{ ft.} \\ KL_y &= (1)(16) = 16.0 \text{ ft.} \end{aligned}$$

Checking the AISC interaction equations:

$$\begin{aligned} f_a &= P_T/A = 81/22.6 = 3.58 \text{ ksi} \\ KL_x/r_x &= (41.23)(12)/7 = 70.68 \\ KL_y/r_y &= (16)(12)/2.47 = 77.70 \\ \therefore F_a &= 15.61 \text{ ksi}, \quad F'_e = 29.9 \text{ ksi} \\ f_b &= M/S_x = (125)(12)/134 = 11.19 \text{ ksi} \\ C_m &= 0.85, \quad F_b = 22 \text{ ksi} \end{aligned}$$

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leq 1.0$$

$$\frac{3.58}{15.61} + \frac{(0.85)(11.19)}{\left(1 - \frac{3.58}{29.9}\right) 22} = 0.72 < 1.0$$

$$\frac{f_a}{0.6F_y} + \frac{f_b}{F_b} = \frac{3.58}{22} + \frac{11.19}{22} = 0.67 < 1.0$$

Upper Column Check:

From Case 3, $P_1 = 33$ kips, $P_2 = 37.5$ kips.

Use the effective length charts contained in the Appendix B to determine K_x .

$$\begin{aligned} I_1/I_2 &= 1.0 \text{ and } L_2/L_T = .75 \\ P_2/P_T &= 33/70.5 = 0.47 \end{aligned}$$

By interpolation from the tables:

$$\begin{aligned} K_1 &= 1.36 \\ KL_x &= (1.36)(42.5) = 57.8 \text{ ft.} \\ KL_y &= 8 \text{ ft.} \end{aligned}$$

Checking the AISC interaction equations:

$$\begin{aligned} f_a &= 33/22.6 = 1.46 \text{ ksi} \\ KL_x/r_x &= (57.8)(12)/7 = 99.08 \\ KL_y/r_y &= (8)(12)/2.47 = 38.87 \end{aligned}$$

$$\begin{aligned} \therefore F_a &= 13.09 \text{ ksi}, \quad F'_e = 15.27 \text{ ksi} \\ f_b &= M/S_x = (117)(12)/134 = 10.48 \text{ ksi} \\ f_a/F_a &= 1.46/13.09 = 0.11 < .15 \end{aligned}$$

Check:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} < 1.0$$

$$= \frac{1.46}{13.09} + \frac{10.48}{22} = .58 < 1.0 \quad \text{o.k.}$$

Use a W16x77

Deflection Controls.

EXAMPLE 20.3.2: Stepped Crane Column Design

Design the column shown in Fig. 20.3.6:

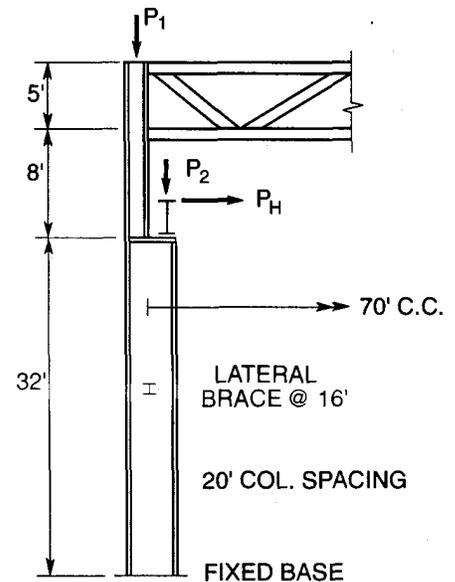


Fig. 20.3.6 Example

Use AISE provisions and A36 steel.

1. Load Cases:

A frame analysis was performed using the AISE loading combinations. The critical moment diagram was obtained from Case 2. See Fig. 20.3.7.

Case 2 = DL + LL + Crane (Lateral and Vertical)

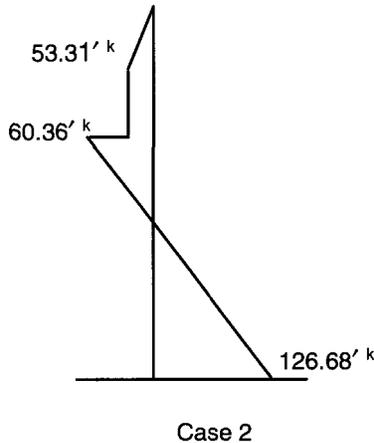


Fig. 20.3.7 Critical Moment Diagram

2. Preliminary Design

Use the strength preliminary design procedures discussed in this guide.

For the upper shaft, $P=31^k$.

Based on the AISC Manual column tables, try a W12x35 section. For the lower shaft the crane load equals 50 kips. Estimate the flange area.

$$0.45 F_y = 16.2 \text{ ksi}$$

$$A_{\text{flange}} = 50/16.2 = 3.09 \text{ in.}^2$$

A W24 section is required for crane clearance.

Try a W24x62, $A_{\text{flange}} = 4.15 \text{ in.}^2$

As an approximation to the moment of inertia for the stepped column, use a weighted average of the moment of inertia for the upper and lower shafts.

$$\frac{(32)(1550) + (10.5)(285)}{42.5} = 1237 \text{ in.}^4$$

Since this average is greater than 984 in.^4 from the previous example, the column should satisfy the $L/240$ deflection requirement. After a final stress check the deflection check can be verified by computer analysis.

3. Stress Check:

The properties of the W12x35 are:

$$A = 10.3 \text{ in.}^2, I_x = 285 \text{ in.}^4$$

$$S_x = 45.6 \text{ in.}^3, r_x = 5.25 \text{ in.}$$

$$r_y = 1.54 \text{ in.}, r_T = 1.74 \text{ in.}$$

For the W24x62:

$$A = 18.2 \text{ in.}^2, I_x = 1550 \text{ in.}^4$$

$$S_x = 131 \text{ in.}^3, r_x = 9.23 \text{ in.}$$

$$r_y = 1.38 \text{ in.}, r_T = 1.71 \text{ in.}$$

$$d/A_f = 5.71$$

Use the effective length charts contained in the Appendix B to determine K_x values. Assume the column base is fixed and the column top is a fixed roller.

$$I_1/I_2 = 285/1550 = 0.18$$

$$L_2/L_T = 32/42.5 = 0.75$$

$$P_2/P_T = 50/81 = 0.62$$

By interpolation from the tables:
 $K_1 = 0.89, K_2 = 1.29$

Lower Column Check:

$$KL_x/r_x = (1.29)(42.5)(12)/9.23 = 71.3$$

$$KL_y/r_y = (1)(16)(12)/1.38 = 139$$

$$\therefore F_a = 7.73 \text{ ksi}, F_e' = 29.38 \text{ ksi}$$

$$f_a = P_T/A = 81/18.2 = 4.45 \text{ ksi}$$

$$f_b = M/S_x = (126.68)(12)/131 = 11.6 \text{ ksi}$$

Determine F_b :

From the moment diagram for the lower shaft (Fig. 20.3.8):

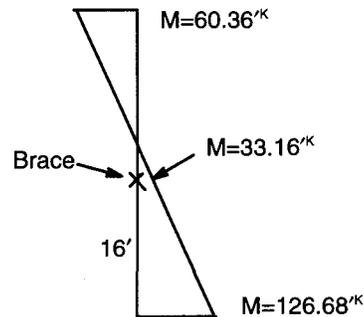


Fig. 20.3.8 Critical Moment Diagram

$$C_b = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2 \leq 2.3$$

$$M_1/M_2 = -33.16/126.68 = -.26$$

$$C_b = 1.75 + 1.05(-.26) + 0.3(-.26)^2 = 1.5$$

$$L/r_T = (16)(12)/1.71 = 112$$

AISC Eq. F1-6 and F1-8 apply.

$$F_b = \left[\frac{2}{3} - \frac{F_y (\ell / r_T)^2}{1530 \times 10^3 C_b} \right] F_y \leq 0.60 F_y$$

$$= \left[\frac{2}{3} - \frac{(36)(112)^2}{(1530 \times 10^3)(1.5)} \right] 36 = 16.88 \text{ ksi}$$

$$F_b = \frac{12 \times 10^3 C_b}{\ell d / A_f} \leq 0.60 F_y$$

$$= \frac{(12000)(1.5)}{(16)(12)(5.71)} = 16.42 \text{ ksi}$$

$\therefore F_b = 16.88 \text{ ksi}$.

Checking the AISC interaction equations:

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0$$

$$\frac{f_a}{0.6 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

$$\frac{4.45}{7.73} + \frac{(0.95)(11.6)}{(1 - 4.45 / 29.38)(16.88)} = 1.35 \quad \text{n.g.}$$

Repeating the above calculation for a W24x68 section yields that a W24x68 is o.k.

Upper Column Check:

$P = 31 \text{ kips}$, $M = 53.31 \text{ ft-kips}$.

The change in I_1/I_2 (change due to W24x68) causes a slight change in the effective length of the upper shaft. Iterating from the effective length tables yields $K1=0.85$

$$KL_x/r_x = (0.85)(42.5)(12)/9.55 = 45.4$$

$$KL_y/r_y = (1)(8)(12)/1.54 = 62$$

$$\therefore F_a = 17.24 \text{ ksi}, F'_e = 72.47 \text{ ksi}$$

$$f_a = P/A = 31/10.3 = 3.01 \text{ ksi}$$

$$f_b = M/S_x = (53.31)(12)/45.6 = 14.03 \text{ ksi}$$

$$F_b = 22 \text{ ksi} \quad (L_u < 8\text{ft})$$

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} \leq 1.0$$

$$= \frac{3.01}{17.24} + \frac{(0.95)(14.03)}{\left(1 - \frac{3.01}{72.47}\right) 22} = 0.81 < 1.0$$

$$\frac{f_a}{0.6 F_y} + \frac{f_b}{F_b} = \frac{3.01}{22} + \frac{14.03}{22} = 0.77 < 1.0$$

Use a W12x35 with a W24x68.

20.4 Economic Considerations

Although it is not possible to provide a clear-cut rule of thumb as to the most economical application of the various crane columns, i.e., bracketed, stepped, or separate crane column, due to differences in shop techniques; it is possible however, to generalize to some degree.

1. The stepped column will be economical if "clean". In fact, for many jobs a "clean" stepped column can prove economical as compared to the bracketed column even for light loads. By "clean" is meant that the column is fabricated without a face channel or extra welded attachments. (See Figure 20.4.1.)

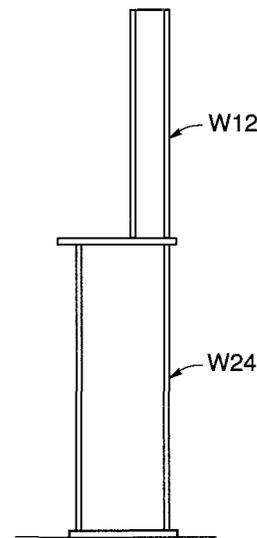


Fig. 20.4.1 "Clean" Column

2. Separate crane columns are economical for heavy cranes. Fabricators favor tying the crane column to the building column with short W shapes acting as a diaphragm as opposed to a lacing system using angles. (See Figure 20.4.2.) Lacing systems are economical as compared to the diaphragm system if the miscellaneous framing pieces are not required. For example, if the building column flange width is equal to the crane column depth, the columns can be laced economically using facing angles. (See Figure 20.4.3.)
3. Bracketed columns are generally most efficient up to bracket loads of 25 kips. Crane reactions between 25 and 50 kips may best be handled by either a bracket column or a stepped column.
4. If the area of one flange of a stepped column multiplied by $.5F_y$ is less than the crane load on the column, a separate crane column should definitely be considered.

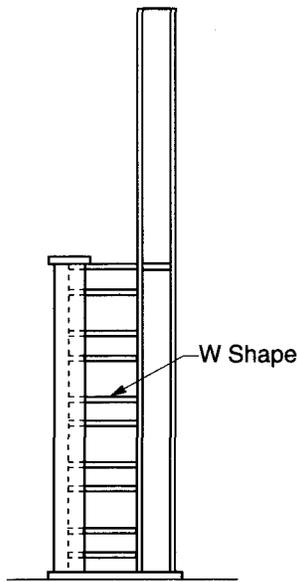
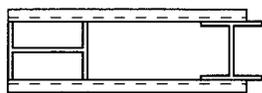
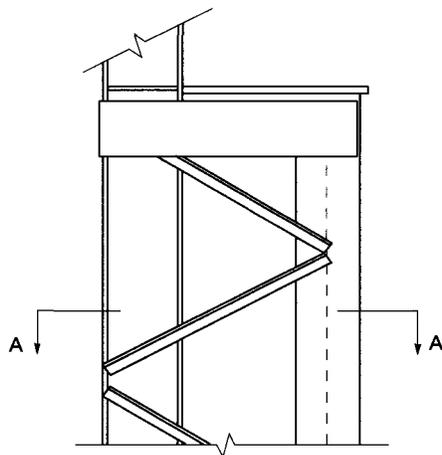


Fig. 20.4.2 Connections with W Shapes



SEC. A-A

Fig. 20.4.3 Laced Column

21. OUTSIDE CRANES

Outside cranes are common in many factories for scrap handling, parts handling and numerous other operations. There are several important aspects of outside crane usage which are unique to that type of crane.

1. The exterior exposure in many climates requires that extra attention be given to painting and general maintenance, material thickness, and the elimination of pockets which would collect moisture.
2. Due to drive aisles, railways and other similar restrictions, exterior cranes often require longer spans than interior cranes. The outside crane has no building columns from which to derive lateral support. Therefore, long, unbraced spans are more common to these installations. Horizontal bracing trusses, wide truss columns or other bracing elements must often be employed to achieve stability.
3. Long spans may dictate that trusses, rather than plate girders or rolled sections, be used for the runway beams. This can have certain advantages including improved stiffness. The disadvantages are clearly the increased depth plus joints which are highly susceptible to fatigue problems. Secondary stresses must be calculated and included in the fatigue analysis for trusses used as crane girders.
4. Another special girder that may be appropriate for use in these long span applications is the trussed girder. This "hybrid" involves the coupling of a girder (top flange) and a truss. The member can develop excellent stiffness characteristics and many times can temporarily support the crane weight even if a truss member is damaged. As with the basic truss, the overall greater depth is a disadvantage.
5. Still another solution to the long span problem may lie in the use of "box" or "semi-box" girders. An excellent reference on this subject was developed by Schlenker.⁽³⁶⁾ These girders have excellent lateral and torsional strength. In addition, the problem associated with off center crane rails is eliminated.
6. Brittle fracture should be considered for cranes operating in low temperature environments.

22. UNDERHUNG CRANES

Underhung cranes in industrial buildings are very common and quite often prove to be economical for special applications. One of the distinct operational advantages that underhung cranes possess is that they can be arranged to provide for trolley transfer from one runway or aisle to another.

Proper provision in the design must be made for handling lateral and impact loads from underhung cranes. The concepts presented in this guide (e.g. load transfer) are, in general, applicable to underhung crane systems. Since these cranes are generally supported by roof members load is not transferred directly to the columns and therefore the column design does not involve the moment distribution problems of the top running crane column. Pay particular attention to the method of hanging the cranes. Fatigue problems with these connections have existed in the past and proper provisions must be made with the hanging connection to guarantee adequate service life.

Hanger systems should provide for vertical adjustment in order to properly adjust the elevation of the runway beam. After the runways are positioned vertically, a lateral antisway brace should be attached at each hanger location. The sway brace prevents the hanger system from flexing perpendicular to the runway. Most hanger systems fatigue at a relatively low stress level if they are allowed to sway. In addition to the lateral antisway braces, longitudinal braces should be installed parallel to the runway beams to prevent sway along the length of the runway. These braces should be placed at approximately 100 foot intervals and at all turns in the runway.

Runway splices can be accomplished in many ways. The splice should allow for a smooth running crane as the wheels transfer from one beam to the next. A typical splice detail is shown in Fig. 22.1.

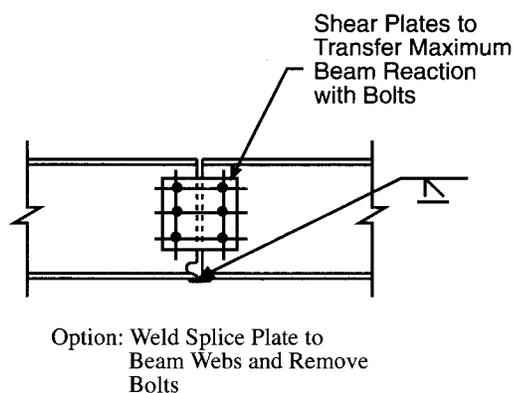


Fig. 22.1 Underhung Crane Beam Splice

Many crane suppliers prefer to supply the runway beams. The building designer must carefully coordinate hanger locations and hanger reactions with the crane supplier. Many times the structure must be designed prior to the selection of the crane system. Hanger locations and reactions must be estimated by the building designer. Hanger reactions can

be calculated from manufacturer's catalogs. Hangers should be provided at a 15 to 20 foot spacing if possible. The deflection limit for underhung crane runway beams due to wheel loads should be limited to span divided by 450.

23. MAINTENANCE AND REPAIR

As stated earlier in this manual, crane buildings require an extra measure of maintenance. Crane rail alignment is especially critical, wear on crane and rail, and potential fatigue problems can result. Crane rails must also be inspected for uneven bearing, to minimize fatigue problems.

If fatigue cracks occur and must be repaired, the repair procedure may create additional problems if proper procedures are not taken. Simple welding of doubler plates, stiffeners or other reinforcement may create a "notch effect" which could be more serious than the original problem. Engineers should use common sense in detailing procedures for repair of fatigue cracks. In particular they should not create a worse fatigue problem with the repair. Referral to Appendix K of the AISC Specifications is essential.

24. SUMMARY AND DESIGN PROCEDURES

Many concepts have been presented in this guide relative to the design and analysis of structural frames for crane buildings. In an effort to optimize design time, the following procedural outline has been developed for the designer.

1. Determine the best geometrical layout for the building in question.
2. Design the crane girders and determine column and frame forces from the crane loadings.
3. Perform preliminary design of the crane columns.
4. Design the roof trusses or roof beams for dead loads and live loads.
5. Determine all loading conditions for which the entire frame must be analyzed.
6. Analyze the frame in question for dead, live, wind and seismic loadings. This analysis should be performed without load sharing from the adjacent frames. Also determine the lateral stiffness of the frame.
7. Analyze the frame (considering load sharing) for crane loadings.
8. Combine moments and forces from the two analyses for subsequent design.
9. Perform the final design of columns, trusses, braces and details.

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APPENDIX A

TABLE 1

| BEAM SECTION | CHANNEL SECTION | TOTAL WEIGHT | AXIS X-X | | | AXIS Y-Y | | |
|--------------|-----------------|--------------|----------|--------|--------|----------|--------|-------|
| | | | I_x | S_x | S_y | Y_1 | I_y | S_z |
| W14x22 | C12x20.7 | 42.7 | 333.9 | 33.41 | 82.92 | 9.994 | 132.48 | 22.08 |
| | C15x33.9 | 55.9 | 372.5 | 34.50 | 111.37 | 10.795 | 318.48 | 42.46 |
| W14x26 | C12x20.7 | 46.7 | 394.7 | 40.09 | 90.79 | 9.844 | 133.44 | 22.24 |
| | C15x33.9 | 59.9 | 440.8 | 41.34 | 120.82 | 10.661 | 319.44 | 42.59 |
| W14x30 | C12x20.7 | 50.7 | 447.1 | 46.71 | 98.25 | 9.571 | 138.77 | 23.12 |
| | C15x33.9 | 63.9 | 498.7 | 48.05 | 129.19 | 10.379 | 324.77 | 43.30 |
| W14x34 | C12x20.7 | 54.7 | 507.1 | 53.50 | 106.01 | 9.478 | 140.63 | 23.43 |
| | C15x33.9 | 67.9 | 565.4 | 54.97 | 138.06 | 10.284 | 326.63 | 43.55 |
| W14x38 | C12x20.7 | 58.7 | 562.4 | 59.91 | 112.59 | 9.386 | 142.31 | 23.71 |
| | C15x33.9 | 71.9 | 627.1 | 61.56 | 145.37 | 10.186 | 328.31 | 43.77 |
| W14x43 | C12x20.7 | 63.7 | 601.0 | 67.37 | 119.67 | 8.919 | 151.57 | 25.26 |
| | C15x33.9 | 76.9 | 667.2 | 68.97 | 152.15 | 9.674 | 337.57 | 45.00 |
| W14x48 | C12x20.7 | 68.7 | 667.1 | 75.38 | 127.73 | 8.849 | 154.67 | 25.77 |
| | C15x33.9 | 81.9 | 740.0 | 77.17 | 160.85 | 9.589 | 340.67 | 45.42 |
| W14x53 | C12x20.7 | 73.7 | 732.9 | 83.31 | 135.61 | 8.797 | 157.79 | 26.29 |
| | C15x33.9 | 86.9 | 812.2 | 85.30 | 169.26 | 9.521 | 343.79 | 45.83 |
| W14x61 | C15x33.9 | 94.9 | 923.6 | 99.43 | 184.71 | 9.289 | 368.66 | 49.15 |
| | MC18x42.7 | 103.7 | 968.8 | 100.53 | 206.04 | 9.637 | 607.66 | 67.51 |
| W14x68 | C15x33.9 | 101.9 | 1023.7 | 110.97 | 196.30 | 9.225 | 375.63 | 50.08 |
| | MC18x42.7 | 110.7 | 1073.4 | 112.19 | 218.10 | 9.568 | 614.63 | 68.29 |
| W14x74 | C15x33.9 | 107.9 | 1108.3 | 120.66 | 205.84 | 9.185 | 381.80 | 50.90 |
| | MC18x42.7 | 116.7 | 1161.8 | 121.99 | 227.98 | 9.523 | 620.80 | 68.97 |
| W14x82 | C15x33.9 | 115.9 | 1211.0 | 132.58 | 217.18 | 9.134 | 389.06 | 51.87 |
| | MC18x42.7 | 124.7 | 1269.0 | 134.07 | 239.65 | 9.464 | 628.06 | 69.78 |
| W16x26 | C12x20.7 | 46.7 | 492.5 | 44.25 | 101.74 | 11.130 | 133.78 | 22.29 |
| | C15x33.9 | 59.9 | 550.5 | 45.66 | 136.48 | 12.055 | 319.78 | 42.63 |
| W16x31 | C12x20.7 | 51.7 | 585.3 | 53.44 | 112.36 | 10.952 | 135.18 | 22.53 |
| | C15x33.9 | 64.9 | 654.4 | 55.07 | 148.84 | 11.882 | 321.18 | 42.82 |
| W16x36 | C12x20.7 | 56.7 | 670.3 | 62.81 | 122.53 | 10.671 | 141.21 | 23.53 |
| | C15x33.9 | 69.9 | 748.3 | 64.60 | 160.04 | 11.584 | 327.21 | 43.62 |
| W16x40 | C12x20.7 | 60.7 | 753.1 | 71.12 | 132.04 | 10.588 | 143.40 | 23.90 |
| | C15x33.9 | 73.9 | 839.5 | 73.05 | 170.69 | 11.491 | 329.40 | 43.92 |
| W16x45 | C12x20.7 | 65.7 | 834.6 | 79.73 | 140.39 | 10.467 | 145.39 | 24.23 |
| | C15x33.9 | 78.9 | 930.1 | 81.93 | 179.66 | 11.352 | 331.39 | 44.18 |

APPENDIX A

TABLE 1

| BEAM SECTION | CHANNEL SECTION | TOTAL WEIGHT | AXIS X-X | | | AXIS Y-Y | | |
|--------------|-----------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | | | I _x | S ₁ | S ₂ | Y ₁ | I _y | S _y |
| W16x50 | C12x20.7 | 70.7 | 918.6 | 88.41 | 149.31 | 10.389 | 147.55 | 24.59 |
| | C15x33.9 | 83.9 | 1022.6 | 90.84 | 189.28 | 11.257 | 333.55 | 44.47 |
| W16x57 | C12x20.7 | 77.7 | 1033.1 | 100.40 | 160.87 | 10.289 | 150.50 | 25.08 |
| | C15x33.9 | 90.9 | 1148.7 | 103.22 | 201.47 | 11.128 | 336.50 | 44.86 |
| W16x67 | C15x33.9 | 100.9 | 1363.6 | 126.53 | 229.06 | 10.776 | 374.41 | 49.92 |
| | MC18x42.7 | 109.7 | 1429.8 | 127.85 | 255.49 | 11.183 | 613.41 | 68.15 |
| W16x77 | C15x33.9 | 110.9 | 1543.4 | 144.67 | 246.89 | 10.668 | 384.10 | 51.21 |
| | MC18x42.7 | 119.7 | 1617.5 | 146.20 | 273.88 | 11.063 | 623.10 | 69.23 |
| W16x89 | C15x33.9 | 122.9 | 1766.7 | 167.06 | 268.71 | 10.575 | 396.19 | 52.82 |
| | MC18x42.7 | 131.7 | 1849.9 | 168.85 | 296.28 | 10.956 | 635.19 | 70.57 |
| W16x100 | C15x33.9 | 133.9 | 1980.8 | 188.04 | 289.77 | 10.534 | 408.00 | 54.40 |
| | MC18x42.7 | 142.7 | 2071.9 | 190.04 | 317.90 | 10.902 | 647.00 | 71.88 |
| W18x35 | C12x20.7 | 55.7 | 785.8 | 65.57 | 131.01 | 11.983 | 136.65 | 22.77 |
| | C15x33.9 | 68.9 | 880.5 | 67.68 | 173.01 | 13.010 | 322.65 | 43.02 |
| W18x40 | C12x20.7 | 60.7 | 908.6 | 76.64 | 143.60 | 11.855 | 138.52 | 23.08 |
| | C15x33.9 | 73.9 | 1016.3 | 78.97 | 187.15 | 12.869 | 324.52 | 43.26 |
| W18x46 | C12x20.7 | 66.7 | 1026.8 | 87.70 | 154.78 | 11.707 | 140.21 | 23.36 |
| | C15x33.9 | 79.9 | 1147.8 | 90.38 | 199.25 | 12.699 | 326.21 | 43.49 |
| W18x50 | C12x20.7 | 70.7 | 1120.4 | 97.36 | 165.65 | 11.508 | 148.99 | 24.83 |
| | C15x33.9 | 83.9 | 1247.7 | 100.04 | 210.82 | 12.471 | 334.99 | 44.66 |
| W18x55 | C12x20.7 | 75.7 | 1224.3 | 107.25 | 175.48 | 11.415 | 151.41 | 25.23 |
| | C15x33.9 | 88.9 | 1361.6 | 110.20 | 221.23 | 12.355 | 337.41 | 44.98 |
| W18x60 | C12x20.7 | 80.7 | 1331.6 | 117.24 | 185.86 | 11.357 | 153.97 | 25.66 |
| | C15x33.9 | 93.9 | 1478.1 | 120.41 | 232.27 | 12.276 | 339.97 | 45.33 |
| W18x65 | C12x20.7 | 85.7 | 1431.6 | 126.77 | 195.05 | 11.292 | 156.32 | 26.05 |
| | C15x33.9 | 98.9 | 1587.1 | 130.23 | 241.83 | 12.186 | 342.32 | 45.64 |
| W18x71 | C12x20.7 | 91.7 | 1543.1 | 137.38 | 205.20 | 11.232 | 159.04 | 26.50 |
| | C15x33.9 | 104.9 | 1708.2 | 141.17 | 252.32 | 12.099 | 345.04 | 46.00 |
| W18x76 | C15x33.9 | 109.9 | 1860.7 | 157.73 | 273.10 | 11.796 | 391.14 | 52.15 |
| | MC18x42.7 | 118.7 | 1950.0 | 159.34 | 303.65 | 12.238 | 630.14 | 70.01 |
| W18x86 | C15x33.9 | 119.9 | 2088.9 | 178.80 | 293.92 | 11.683 | 402.51 | 53.66 |
| | MC18x42.7 | 128.7 | 2187.3 | 180.62 | 325.01 | 12.109 | 641.51 | 71.27 |
| W18x97 | C15x33.9 | 130.9 | 2341.2 | 201.79 | 316.89 | 11.601 | 415.36 | 55.38 |
| | MC18x42.7 | 139.7 | 2448.9 | 203.84 | 348.54 | 12.013 | 654.36 | 72.70 |

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TABLE 1

| BEAM SECTION | CHANNEL SECTION | TOTAL WEIGHT | AXIS X-X | | | AXIS Y-Y | | |
|--------------|-----------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | | | I _x | S ₁ | S ₂ | Y ₁ | I _y | S _y |
| W18x106 | C15x33.9 | 139.9 | 2526.6 | 218.89 | 333.01 | 11.542 | 425.05 | 56.67 |
| | MC18x42.7 | 148.7 | 2641.2 | 221.16 | 364.91 | 11.942 | 664.05 | 73.78 |
| W18x119 | C15x33.9 | 152.9 | 2841.3 | 247.16 | 360.85 | 11.496 | 441.27 | 58.83 |
| | MC18x42.7 | 161.7 | 2966.1 | 249.72 | 393.26 | 11.877 | 680.27 | 75.58 |
| W21x44 | C12x20.7 | 64.7 | 1254.4 | 92.97 | 168.39 | 13.492 | 139.29 | 23.21 |
| | C15x33.9 | 77.9 | 1408.5 | 96.19 | 219.51 | 14.643 | 325.29 | 43.37 |
| W21x50 | C12x20.7 | 70.7 | 1418.6 | 106.31 | 182.62 | 13.344 | 141.41 | 23.56 |
| | C15x33.9 | 83.9 | 1589.3 | 109.87 | 234.95 | 14.465 | 327.41 | 43.65 |
| W21x57 | C12x20.7 | 77.7 | 1629.2 | 123.11 | 200.90 | 13.232 | 144.25 | 24.04 |
| | C15x33.9 | 90.0 | 1818.8 | 127.01 | 254.71 | 14.319 | 330.25 | 44.03 |
| W21x62 | C12x20.7 | 82.7 | 1800.9 | 138.40 | 218.02 | 13.011 | 157.67 | 26.27 |
| | C15x33.9 | 95.9 | 1999.9 | 142.27 | 272.75 | 14.057 | 343.67 | 45.82 |
| W21x68 | C12x20.7 | 88.7 | 1963.8 | 151.83 | 231.63 | 12.934 | 161.28 | 26.88 |
| | C15x33.9 | 101.9 | 2175.9 | 156.00 | 287.01 | 13.948 | 347.28 | 46.30 |
| W21x73 | C12x20.7 | 93.7 | 2101.6 | 163.26 | 242.97 | 12.872 | 164.19 | 27.36 |
| | C15x33.9 | 106.9 | 2324.5 | 167.71 | 298.76 | 13.859 | 350.19 | 46.69 |
| W21x83 | C12x20.7 | 103.7 | 2352.6 | 184.10 | 263.36 | 12.778 | 169.58 | 28.26 |
| | C15x33.9 | 116.9 | 2593.9 | 189.09 | 319.74 | 13.717 | 355.58 | 47.41 |
| W21x93 | C12x20.7 | 113.7 | 2617.3 | 205.99 | 284.60 | 12.705 | 175.26 | 29.21 |
| | C15x33.9 | 126.9 | 2876.4 | 211.56 | 341.46 | 13.596 | 361.26 | 48.16 |
| W21x101 | MC18x42.7 | 143.7 | 3369.7 | 245.48 | 416.88 | 13.726 | 677.75 | 75.30 |
| W21x111 | MC18x42.7 | 153.7 | 3662.5 | 268.75 | 439.56 | 13.627 | 691.01 | 76.77 |
| W21x122 | MC18x42.7 | 164.7 | 3985.0 | 294.20 | 464.19 | 13.545 | 706.16 | 78.46 |
| W21x132 | MC18x42.7 | 174.7 | 4280.5 | 317.40 | 486.75 | 13.485 | 720.04 | 80.00 |
| W21x147 | MC18x42.7 | 189.7 | 4739.9 | 353.08 | 521.68 | 13.424 | 741.62 | 82.40 |
| W24x55 | C12x20.7 | 75.7 | 1919.4 | 128.89 | 214.20 | 14.891 | 143.46 | 23.91 |
| | C15x33.9 | 88.9 | 2152.9 | 133.51 | 274.41 | 16.124 | 329.46 | 43.92 |
| W24x62 | C12x20.7 | 82.7 | 2157.5 | 146.35 | 232.48 | 14.741 | 146.15 | 24.35 |
| | C15x33.9 | 95.9 | 2411.9 | 151.39 | 293.82 | 15.931 | 332.15 | 44.28 |
| W24x68 | C12x20.7 | 88.7 | 2443.7 | 168.21 | 257.64 | 14.527 | 164.12 | 27.35 |
| | C15x33.9 | 101.9 | 2712.7 | 173.13 | 320.58 | 15.668 | 350.12 | 46.68 |
| W24x76 | C12x20.7 | 96.7 | 2746.9 | 190.39 | 281.03 | 14.427 | 170.17 | 28.36 |
| | C15x33.9 | 109.9 | 3036.4 | 195.62 | 345.13 | 15.522 | 356.17 | 47.48 |

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TABLE 1

| BEAM SECTION | CHANNEL SECTION | TOTAL WEIGHT | AXIS X-X | | | AXIS Y-Y | | |
|--------------|-----------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | | | I _x | S _x | S _y | Y ₁ | I _y | S _y |
| W24x84 | C12x20.7 | 104.7 | 3026.9 | 210.91 | 301.76 | 14.351 | 176.09 | 29.34 |
| | C15x33.9 | 117.9 | 3335.4 | 216.56 | 366.59 | 15.401 | 362.09 | 48.27 |
| W24x94 | C12x20.7 | 114.7 | 3390.2 | 237.56 | 328.47 | 14.270 | 183.31 | 30.55 |
| | C15x33.9 | 127.9 | 3721.0 | 243.72 | 794.06 | 15.267 | 369.31 | 49.24 |
| W24x104 | MC18x42.7 | 146.7 | 4319.7 | 280.24 | 474.91 | 15.414 | 683.54 | 75.94 |
| W24x117 | MC18x42.7 | 158.7 | 4807.2 | 314.87 | 509.10 | 15.267 | 702.54 | 78.06 |
| W24x131 | MC18x42.7 | 173.7 | 5366.1 | 354.13 | 548.83 | 15.152 | 723.94 | 80.43 |
| W24x146 | MC18x42.7 | 188.7 | 5993.5 | 397.54 | 592.63 | 15.076 | 748.99 | 83.22 |
| W24x162 | MX18x42.7 | 204.7 | 6642.1 | 442.14 | 637.00 | 15.022 | 775.05 | 86.11 |
| W27x84 | C15x33.9 | 117.9 | 4047.7 | 237.11 | 403.19 | 17.070 | 367.69 | 49.02 |
| | MC18x42.7 | 126.7 | 4255.4 | 240.27 | 450.32 | 17.710 | 606.69 | 67.41 |
| W27x94 | C15x33.9 | 127.9 | 4530.9 | 267.82 | 435.55 | 16.917 | 376.89 | 50.25 |
| | MC18x42.7 | 136.7 | 4756.2 | 271.24 | 483.59 | 17.534 | 615.89 | 68.43 |
| W27x102 | C15x33.9 | 135.9 | 4919.2 | 292.38 | 461.23 | 16.824 | 384.47 | 51.26 |
| | MC18x42.7 | 144.7 | 5157.8 | 596.00 | 509.91 | 17.424 | 623.47 | 69.27 |
| W27x114 | C15x33.9 | 147.9 | 5437.4 | 325.92 | 494.02 | 16.683 | 394.13 | 52.55 |
| | MC18x42.7 | 156.7 | 5693.9 | 329.93 | 543.20 | 17.257 | 633.13 | 70.34 |
| W27x146 | MC18x42.7 | 188.7 | 7354.2 | 440.34 | 660.82 | 16.701 | 775.28 | 86.14 |
| W27x161 | MC18x42.7 | 203.7 | 8069.9 | 486.07 | 705.55 | 16.602 | 802.01 | 89.11 |
| W27x178 | MC18x42.7 | 220.7 | 8839.2 | 535.00 | 753.02 | 16.521 | 831.09 | 92.34 |
| W30x99 | C15x33.9 | 132.9 | 5542.8 | 299.50 | 480.17 | 18.506 | 378.71 | 50.49 |
| | MC18x42.7 | 141.7 | 5825.0 | 303.77 | 533.21 | 19.175 | 617.71 | 68.63 |
| W30x108 | C15x33.9 | 141.9 | 6067.3 | 329.95 | 512.36 | 18.388 | 387.79 | 51.70 |
| | MC18x42.7 | 150.7 | 6366.5 | 334.45 | 566.20 | 19.035 | 626.79 | 69.64 |
| W30x116 | C15x33.9 | 149.9 | 6593.0 | 360.23 | 544.52 | 18.301 | 396.88 | 52.91 |
| | MC18x42.7 | 158.7 | 6907.8 | 364.91 | 599.11 | 18.929 | 635.88 | 70.65 |
| W30x124 | C15x33.9 | 157.9 | 7053.7 | 386.80 | 571.88 | 18.235 | 405.10 | 54.01 |
| | MC18x42.7 | 166.7 | 7382.0 | 391.69 | 627.00 | 18.846 | 644.10 | 71.56 |
| W30x132 | C15x33.9 | 165.9 | 7496.4 | 412.67 | 597.58 | 18.165 | 412.71 | 55.02 |
| | MC18x42.7 | 174.7 | 7837.7 | 417.82 | 653.05 | 18.758 | 651.71 | 72.41 |
| W30x173 | MC18x42.7 | 215.7 | 10427.2 | 574.19 | 819.10 | 18.159 | 852.63 | 94.73 |

APPENDIX A

TABLE 1

| BEAM SECTION | CHANNEL SECTION | TOTAL WEIGHT | AXIS X-X | | | AXIS Y-Y | | |
|--------------|-----------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | | | I _x | S ₁ | S ₂ | Y ₁ | I _t | S _t |
| W30x191 | MC18x42.7 | 233.7 | 11475.9 | 634.90 | 879.05 | 18.075 | 889.95 | 98.88 |
| W30x211 | MC18x42.7 | 253.7 | 12640.7 | 701.84 | 944.80 | 18.010 | 931.66 | 118.35 |
| W33x118 | C15x33.9 | 151.9 | 7898.2 | 394.75 | 596.00 | 20.007 | 408.29 | 54.43 |
| | MC18x42.7 | 160.7 | 8280.0 | 400.13 | 656.25 | 20.692 | 647.29 | 71.92 |
| W33x130 | C15x33.9 | 163.9 | 8789.0 | 442.11 | 645.76 | 19.879 | 423.64 | 56.48 |
| | MC18x42.7 | 172.7 | 9194.7 | 447.76 | 707.01 | 20.534 | 662.64 | 73.62 |
| W33x141 | C15x33.9 | 174.9 | 9592.7 | 484.68 | 689.70 | 19.791 | 437.78 | 58.37 |
| | MC18x42.7 | 183.7 | 10018.8 | 490.60 | 751.67 | 20.421 | 676.78 | 75.19 |
| W33x152 | C15x33.9 | 185.9 | 10342.4 | 524.31 | 730.17 | 19.725 | 450.99 | 60.13 |
| | MC18x42.7 | 194.7 | 10786.5 | 530.48 | 792.73 | 20.333 | 689.99 | 76.66 |
| W36x33.9 | C15x33.9 | 168.9 | 10218.7 | 480.60 | 695.73 | 21.262 | 427.34 | 56.97 |
| | MC18x42.7 | 177.7 | 10696.0 | 487.19 | 761.52 | 21.954 | 666.34 | 74.03 |
| W36x150 | C15x33.9 | 183.9 | 11542.4 | 545.73 | 764.41 | 21.150 | 449.51 | 59.93 |
| | MC18x42.7 | 192.7 | 12050.6 | 552.61 | 831.45 | 21.806 | 688.51 | 76.50 |
| W36x160 | C15x33.9 | 193.9 | 12317.7 | 584.17 | 803.80 | 21.085 | 461.88 | 61.58 |
| | MC18x42.7 | 202.7 | 12843.2 | 591.27 | 871.38 | 21.721 | 700.88 | 77.87 |
| W36x170 | C15x33.9 | 203.9 | 13098.8 | 623.01 | 842.63 | 21.024 | 474.59 | 63.27 |
| | MC18x42.7 | 212.7 | 13641.6 | 630.41 | 910.60 | 21.639 | 713.59 | 79.28 |
| W36x182 | C15x33.9 | 215.9 | 13979.5 | 667.25 | 885.95 | 20.950 | 488.12 | 65.08 |
| | MC18x42.7 | 224.7 | 14541.0 | 675.03 | 954.20 | 21.541 | 727.12 | 80.79 |
| W36x194 | C15x33.9 | 227.9 | 14826.6 | 709.36 | 927.32 | 20.901 | 501.70 | 66.89 |
| | MC18x42.7 | 236.7 | 15405.1 | 717.49 | 995.85 | 21.470 | 740.70 | 82.30 |

APPENDIX A

TABLE 2

| BEAM SECTION | CHANNEL SECTION | COMPOSITE SECTION | | MAXIMUM SPAN (FT.) FOR $F_b = 0.6F_y$ | |
|--------------|-----------------|-------------------|---------|---|----------------|
| | | r_T | d/A_T | $F_y = 36$ ksi | $F_y = 50$ ksi |
| W14x22 | C12x20.7 | 4.130 | 1.80 | 25.63 | 18.45 |
| | C15x33.9 | 5.231 | 1.21 | 38.09 | 27.42 |
| W14x26 | C12x20.7 | 4.033 | 1.73 | 26.75 | 19.26 |
| | C15x33.9 | 5.144 | 1.18 | 39.05 | 28.11 |
| W14x30 | C12x20.7 | 3.998 | 1.62 | 28.45 | 20.49 |
| | C15x33.9 | 5.086 | 1.13 | 40.80 | 29.37 |
| W14x34 | C12x20.7 | 3.918 | 1.55 | 29.73 | 21.40 |
| | C15x33.9 | 5.006 | 1.10 | 41.94 | 30.20 |
| W14x38 | C12x20.7 | 3.854 | 1.50 | 30.82 | 22.19 |
| | C15x33.9 | 4.941 | 1.07 | 42.93 | 30.91 |
| W14x43 | C12x20.7 | 3.831 | 1.35 | 34.29 | 24.69 |
| | C15x33.9 | 4.876 | 0.99 | 46.74 | 33.65 |
| W14x48 | C12x20.7 | 3.772 | 1.29 | 35.75 | 25.74 |
| | C15x33.9 | 4.807 | 0.96 | 48.08 | 34.62 |
| W14x53 | C12x20.7 | 3.718 | 1.24 | 37.19 | 26.77 |
| | C15x33.9 | 4.743 | 0.93 | 49.39 | 35.56 |
| W14x61 | C15x33.9 | 4.740 | 0.87 | 53.15 | 38.27 |
| | MC18x42.7 | 5.648 | 0.75 | 61.49 | 44.27 |
| W14x68 | C15x33.9 | 4.675 | 0.84 | 55.09 | 39.67 |
| | MC18x42.7 | 5.567 | 0.73 | 63.34 | 45.60 |
| W14x74 | C15x33.9 | 4.622 | 0.81 | 56.76 | 40.87 |
| | MC18x42.7 | 5.502 | 0.71 | 64.93 | 46.75 |
| W14x82 | C15x33.9 | 4.570 | 0.78 | 58.60 | 42.19 |
| | MC18x42.7 | 5.435 | 0.69 | 66.68 | 48.01 |
| W14x90 | MC18x42.7 | 5.664 | 0.63 | 73.29 | 52.77 |
| W14x99 | MC18x42.7 | 5.612 | 0.60 | 75.92 | 54.66 |
| W14x109 | MC18x42.7 | 5.558 | 0.58 | 78.86 | 56.78 |
| W14x120 | MC18x42.7 | 5.510 | 0.56 | 81.83 | 58.91 |
| W14x132 | MC18x42.7 | 5.460 | 0.54 | 85.07 | 61.25 |
| W16x26 | C12x20.7 | 4.092 | 1.99 | 23.15 | 16.66 |
| | C15x33.9 | 5.193 | 1.35 | 34.11 | 24.56 |

APPENDIX A

TABLE 2

| BEAM SECTION | CHANNEL SECTION | COMPOSITE SECTION | | MAXIMUM SPAN (FT.) FOR $F_b = 0.6F_y$ | |
|--------------|-----------------|-------------------|---------|---|----------------|
| | | r_T | d/A_f | $F_y = 36$ ksi | $F_y = 50$ ksi |
| W16x31 | C12x20.7 | 3.983 | 1.89 | 24.40 | 17.57 |
| | C15x33.9 | 5.091 | 1.31 | 35.23 | 25.37 |
| W16x36 | C12x20.7 | 3.940 | 1.77 | 26.08 | 18.77 |
| | C15x33.9 | 5.024 | 1.25 | 36.91 | 26.57 |
| W16x40 | C12x20.7 | 3.860 | 1.69 | 27.34 | 19.68 |
| | C15x33.9 | 4.941 | 1.21 | 38.06 | 27.40 |
| W16x45 | C12x20.7 | 3.800 | 1.63 | 28.39 | 20.44 |
| | C15x33.9 | 4.876 | 1.18 | 39.02 | 28.09 |
| W16x50 | C12x20.7 | 3.740 | 1.56 | 29.50 | 21.24 |
| | C15x33.9 | 4.810 | 1.15 | 40.05 | 28.83 |
| W16x57 | C12x20.7 | 3.668 | 1.49 | 30.97 | 22.30 |
| | C15x33.9 | 4.728 | 1.11 | 41.40 | 29.80 |
| W16x67 | C15x33.9 | 4.725 | 0.99 | 46.39 | 33.40 |
| | MC18x42.7 | 5.622 | 0.86 | 53.54 | 38.55 |
| W16x77 | C15x33.9 | 4.647 | 0.95 | 48.66 | 35.03 |
| | MC18x42.7 | 5.523 | 0.83 | 55.71 | 40.11 |
| W16x89 | C15x33.9 | 4.562 | 0.90 | 51.36 | 36.98 |
| | MC18x42.7 | 5.414 | 0.79 | 58.32 | 41.99 |
| W16x100 | C15x33.9 | 4.491 | 0.85 | 53.91 | 38.81 |
| | MC18x42.7 | 5.319 | 0.76 | 60.77 | 43.75 |
| W18x35 | C12x20.7 | 3.976 | 2.08 | 22.24 | 16.01 |
| | C15x33.9 | 5.078 | 1.44 | 31.99 | 23.03 |
| W18x40 | C12x20.7 | 3.870 | 1.96 | 23.54 | 16.95 |
| | C15x33.9 | 4.973 | 1.39 | 33.18 | 23.89 |
| W18x46 | C12x20.7 | 3.791 | 1.88 | 24.62 | 17.73 |
| | C15x33.9 | 4.892 | 1.35 | 34.17 | 24.60 |
| W18x50 | C12x20.7 | 3.791 | 1.76 | 26.25 | 18.90 |
| | C15x33.9 | 4.851 | 1.29 | 35.82 | 25.79 |
| W18x55 | C12x20.7 | 3.738 | 1.69 | 27.27 | 19.63 |
| | C15x33.9 | 4.790 | 1.25 | 36.77 | 26.47 |
| W18x60 | C12x20.7 | 3.684 | 1.63 | 28.34 | 20.40 |
| | C15x33.9 | 4.727 | 1.22 | 37.77 | 27.20 |
| W18x65 | C12x20.7 | 3.642 | 1.58 | 29.27 | 21.07 |
| | C15x33.9 | 4.676 | 1.19 | 38.64 | 27.82 |

APPENDIX A

TABLE 2

| BEAM SECTION | CHANNEL SECTION | COMPOSITE SECTION | | MAXIMUM SPAN (FT.) FOR $F_b = 0.6F_y$ | |
|--------------|-----------------|-------------------|---------|---|----------------|
| | | r_T | d/A_r | $F_y = 36$ ksi | $F_y = 50$ ksi |
| W18x71 | C12x20.7 | 3.599 | 1.52 | 30.30 | 21.81 |
| | C15x33.9 | 4.623 | 1.16 | 39.60 | 28.51 |
| W18x76 | C15x33.9 | 4.732 | 1.06 | 43.44 | 31.28 |
| | MC18x42.7 | 5.598 | 0.92 | 49.87 | 35.91 |
| W18x86 | C15x33.9 | 4.664 | 1.01 | 45.58 | 32.81 |
| | MC18x42.7 | 5.508 | 0.89 | 51.94 | 37.40 |
| W18x97 | C15x33.9 | 4.596 | 0.96 | 47.92 | 34.50 |
| | MC18x42.7 | 5.417 | 0.85 | 54.21 | 39.03 |
| W18x106 | C15x33.9 | 4.554 | 0.93 | 49.58 | 35.69 |
| | MC18x42.7 | 5.358 | 0.82 | 55.82 | 40.19 |
| W18x119 | C15x33.9 | 4.488 | 0.88 | 52.34 | 37.68 |
| | MC18x42.7 | 5.264 | 0.79 | 58.50 | 42.12 |
| W21x44 | C12x20.7 | 3.930 | 2.32 | 19.92 | 14.79 |
| | C15x33.9 | 5.024 | 1.63 | 28.32 | 20.39 |
| W21x50 | C12x20.7 | 3.841 | 2.20 | 21.01 | 15.13 |
| | C15x33.9 | 4.933 | 1.57 | 29.33 | 21.12 |
| W21x57 | C12x20.7 | 3.733 | 2.06 | 22.45 | 16.16 |
| | C15x33.9 | 4.819 | 1.50 | 30.67 | 22.08 |
| W21x62 | C12x20.7 | 3.759 | 1.90 | 24.28 | 17.48 |
| | C15x33.9 | 4.782 | 1.42 | 32.52 | 23.41 |
| W21x68 | C12x20.7 | 3.704 | 1.82 | 25.41 | 18.29 |
| | C15x33.9 | 4.714 | 1.37 | 33.59 | 24.19 |
| W21x73 | C12x20.7 | 3.664 | 1.76 | 26.30 | 18.93 |
| | C15x33.9 | 4.664 | 1.34 | 34.44 | 24.79 |
| W21x83 | C12x20.7 | 3.602 | 1.66 | 27.86 | 20.06 |
| | C15x33.9 | 4.582 | 1.28 | 35.91 | 25.86 |
| W21x93 | C12x20.7 | 3.548 | 1.57 | 29.42 | 21.18 |
| | C15x33.9 | 4.506 | 1.23 | 37.40 | 26.93 |
| W21x101 | MC18x42.7 | 5.496 | 0.97 | 47.61 | 34.28 |
| W21x111 | MC18x42.7 | 5.434 | 0.93 | 49.32 | 35.51 |
| W21x122 | MC18x42.7 | 5.369 | 0.90 | 51.24 | 36.89 |
| W21x132 | MC18x42.7 | 5.316 | 0.87 | 52.93 | 38.11 |
| W21x147 | MC18x42.7 | 5.242 | 0.83 | 55.50 | 39.96 |

APPENDIX A

TABLE 2

| BEAM SECTION | CHANNEL SECTION | COMPOSITE SECTION | | MAXIMUM SPAN (FT.) FOR $F_b = 0.6F_y$ | |
|--------------|-----------------|-------------------|---------|---|----------------|
| | | r_T | d/A_f | $F_y = 36$ ksi | $F_y = 50$ ksi |
| W21x147 | MC18x42.7 | 5.242 | 0.83 | 55.50 | 39.96 |
| W24x55 | C12x20.7 | 3.860 | 2.47 | 18.68 | 14.52 |
| | C15x33.9 | 4.940 | 1.77 | 26.06 | 18.77 |
| W24x62 | C12x20.7 | 3.777 | 2.34 | 19.74 | 14.21 |
| | C15x33.9 | 4.851 | 1.71 | 27.06 | 19.48 |
| W24x68 | C12x20.7 | 3.805 | 2.11 | 21.85 | 15.73 |
| | C15x33.9 | 4.798 | 1.58 | 29.17 | 21.00 |
| W24x76 | C12x20.7 | 3.734 | 1.98 | 23.34 | 16.80 |
| | C15x33.9 | 4.707 | 1.51 | 30.59 | 22.03 |
| W24x84 | C12x20.7 | 3.675 | 1.87 | 24.75 | 17.82 |
| | C15x33.9 | 4.628 | 1.44 | 31.94 | 23.00 |
| W24x94 | C12x20.7 | 3.615 | 1.75 | 26.39 | 19.00 |
| | C15x33.9 | 4.543 | 1.38 | 33.52 | 24.13 |
| W24x104 | MC18x42.7 | 5.553 | 1.10 | 41.86 | 30.14 |
| W24x117 | MC18x42.7 | 5.470 | 1.05 | 43.99 | 31.67 |
| W24x131 | MC18x42.7 | 5.387 | 0.99 | 46.31 | 33.34 |
| W24x146 | MC18x42.7 | 5.300 | 0.94 | 48.99 | 35.27 |
| W24x162 | MC18x42.7 | 5.223 | 0.89 | 51.67 | 37.20 |
| W27x84 | C15x33.9 | 4.744 | 1.65 | 27.89 | 20.08 |
| | MC18x42.7 | 5.654 | 1.43 | 32.34 | 23.28 |
| W27x94 | C15x33.9 | 4.653 | 1.56 | 29.49 | 21.23 |
| | MC18x42.7 | 5.543 | 1.36 | 33.90 | 24.40 |
| W27x102 | C15x33.9 | 4.587 | 1.50 | 30.77 | 22.15 |
| | MC18x42.7 | 5.460 | 1.31 | 35.15 | 25.31 |
| W27x114 | C15x33.9 | 4.516 | 1.43 | 32.31 | 23.26 |
| | MC18x42.7 | 5.368 | 1.26 | 36.65 | 26.39 |
| W27x146 | MC18x42.7 | 5.438 | 1.06 | 43.61 | 31.40 |
| W27x161 | MC18x42.7 | 5.376 | 1.01 | 45.80 | 32.97 |
| W27x178 | MC18x42.7 | 5.320 | 0.96 | 48.10 | 34.63 |
| W30x99 | C15x33.9 | 4.725 | 1.77 | 26.13 | 18.81 |
| | MC18x42.7 | 5.613 | 1.53 | 30.14 | 21.70 |

APPENDIX A

TABLE 2

| BEAM SECTION | CHANNEL SECTION | COMPOSITE SECTION | | MAXIMUM SPAN (FT.) FOR $F_b = 0.6F_y$ | |
|--------------|-----------------|-------------------|---------|---|----------------|
| | | r_T | d/A_f | $F_y = 36$ ksi | $F_y = 50$ ksi |
| W30x108 | C15x33.9 | 4.651 | 1.68 | 27.44 | 19.76 |
| | MC18x42.7 | 5.521 | 1.47 | 31.43 | 22.63 |
| W30x116 | C15x33.9 | 4.584 | 1.61 | 28.74 | 20.69 |
| | MC18x42.7 | 5.435 | 1.41 | 32.70 | 23.55 |
| W30x124 | C15x33.9 | 4.530 | 1.54 | 29.89 | 21.52 |
| | MC18x42.7 | 5.364 | 1.36 | 33.83 | 24.36 |
| W30x132 | C15x33.9 | 4.486 | 1.49 | 30.91 | 22.25 |
| | MC18x42.7 | 5.306 | 1.32 | 34.83 | 25.08 |
| W30x173 | MC18x42.7 | 5.463 | 1.08 | 42.80 | 30.81 |
| W30x191 | MC18x42.7 | 5.408 | 1.02 | 45.24 | 32.57 |
| W30x211 | MC18x42.7 | 5.357 | 0.96 | 47.87 | 34.47 |
| W33x118 | C15x33.9 | 4.703 | 1.80 | 25.68 | 18.49 |
| | MC18x42.7 | 5.539 | 1.57 | 29.31 | 21.10 |
| W33x130 | C15x33.9 | 4.625 | 1.69 | 27.37 | 19.70 |
| | MC18x42.7 | 5.433 | 1.49 | 30.97 | 22.30 |
| W33x141 | C15x33.9 | 4.562 | 1.60 | 28.89 | 20.80 |
| | MC18x42.7 | 5.346 | 1.42 | 32.47 | 23.38 |
| W33x152 | C15x33.9 | 4.511 | 1.52 | 30.27 | 21.79 |
| | MC18x42.7 | 5.274 | 1.36 | 33.83 | 24.35 |
| W36x135 | C15x33.9 | 4.693 | 1.85 | 24.98 | 17.98 |
| | MC18x42.7 | 5.498 | 1.63 | 28.34 | 20.69 |
| W36x150 | C15x33.9 | 4.602 | 1.70 | 27.09 | 19.50 |
| | MC18x42.7 | 5.372 | 1.52 | 30.42 | 21.90 |
| W36x150 | C15x33.9 | 4.561 | 1.64 | 28.22 | 20.32 |
| | MC18x42.7 | 5.311 | 1.46 | 31.54 | 22.70 |
| W36x170 | C15x33.9 | 4.523 | 1.57 | 29.36 | 21.14 |
| | MC18x42.7 | 5.255 | 1.41 | 32.65 | 23.51 |
| W36x182 | C15x33.9 | 4.490 | 1.51 | 30.51 | 21.96 |
| | MC18x42.7 | 5.204 | 1.36 | 33.79 | 24.33 |
| W36x194 | C15x33.9 | 4.459 | 1.46 | 31.65 | 22.79 |
| | MC18x42.7 | 5.155 | 1.32 | 34.92 | 25.14 |
| W36x210 | MC18x42.7 | 5.100 | 1.27 | 36.35 | 26.17 |

Appendix B

CALCULATION OF EFFECTIVE LENGTHS OF STEPPED COLUMNS

KRISHNA M. AGRAWAL AND ANDREW P. STAFIEJ

Travelling cranes are frequently used to move heavy loads in industrial buildings. To accomplish a general movement, the crane traverses a crane bridge, which in turn moves on rails along the length of the building, supported by the main building columns.

Designers frequently choose stepped columns, with the wider lower section serving a dual purpose: (a) to support the crane rail and (b) to provide the necessary strength to support the extra load from the crane. The design of the stepped columns is time-consuming and complicated. Effective lengths, which must be calculated for each segment, depend upon the following: the end fixity types at the two ends, the ratio of segment lengths (l_1/l_2), the ratio of the segment inertias I_1/I_2 , and the ratio of the applied axial loads (P_1/P_2) applied at the top of the column and at the stepped levels.

Various cases of end fixities are encountered in practice (Fig. 1). Anderson and Woodward¹ have presented equations for five end-fixity types to be used in calculating effective lengths. These types are: (1) Pin-Pin (2) Fix-Free, (3) Fix-Pin, (4) Fix-Slider, and (5) Fix-Fix. Two other cases which have not been dealt with previously are: (6) Pin-Fix and (7) Pin-Slider (Fig. 1). Industrial building frames are often designed as pinned at the bottom, supporting a deep roof truss at the top which provides for a fix or slider end condition.

The characteristic equation in Ref. 1 for case (5) when $P_2 = 0$ [Eq. (A-15) and FUNCTION FC5(x)] appears to be in error. This technical note is intended to correct the equation for the end-fixity case (5) and to extend the directory of characteristic equations for end-fixities to include two additional cases: (6) Pin-Fix and (7) Pin-Slider. The derivation of the equations is omitted in this paper, since the process has been adequately described in Ref. 1. The nomenclature of Ref. 1 is used throughout to maintain continuity.

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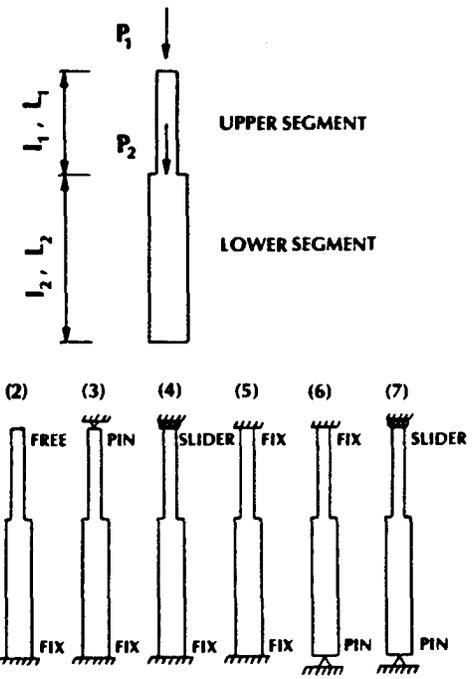


Fig. 1. End condition types.

CHARACTERISTIC EQUATIONS

The parameters in the equations have the following definitions:

$$IR = I_1 / I_2; LR = l_1 / l_2; PR = P_1 / P_2$$

$$Z = Y_1 / l_1; BZ = Y_2 / l_2$$

$$\beta = B = (l_2 / l_1) \times \sqrt{(I_1 / I_2)[1 + (P_2 / P_1)]}$$

$$Y_1^2 = P_1 / EI_1; Y_2^2 = (P_1 + P_2) / EI_2$$

Finding the lowest root of the characteristic equation $Z = ZRT$ allows the calculation of buckling load

$$P_{1cr} = \left(\frac{ZRT}{l_1} \right)^2 \times EI_1$$

$$(P_1 + P_2)_{cr} = \left(\frac{ZRT \times \beta}{l_2} \right)^2 \times EI_2$$

Equating these critical loads to the Euler buckling formula $P_{cr} = \pi^2 EI / L^2$, one obtains:

$$l_{1(eff)} = \pi l_1 / ZRT$$

$$l_{2(eff)} = \pi l_2 / (\beta Z)$$

These effective lengths are used in the AISC interaction formula for designing beam columns.

The variable definitions are:

- I_1, I_2 Moments of inertia of upper and lower segments, respectively
 P_1, P_2 Applied axial loads at the top and at step level
 P_T Total column axial load = $(P_1 + P_2)$
 l_1, l_2 Lengths of upper and lower segments, respectively
 L_T Total column length = $(l_1 + l_2)$
 $l_{1(eff)}, l_{2(eff)}$ Effective lengths of the upper and lower segments for Euler buckling formula, respectively
 K_1, K_2 Effective length factors for upper and lower segments, respectively, with the following definition:

$$K_1 = l_{1(eff)} / (l_1 + l_2)$$

$$K_2 = l_{2(eff)} / (l_1 + l_2)$$

Case 5—Fix-Fix [corrected characteristic equation to replace Eq. (A-15) in Ref. 1]:

$$\begin{aligned} \text{c. } P_2 = 0 \\ [\text{COS}(Z) - \text{COS}(BZ)]\{Z[(1 + LR) / LR] \times \text{SIN}(Z) \\ + \text{COS}(Z) - \text{COS}(BZ)\} + [LR \times \text{SIN}(Z) \\ - Z(1 + LR)\text{COS}(Z) + \text{SIN}(BZ) / B][\text{SIN}(Z) / \\ LR + B \times \text{sin}(BZ)] = 0 \end{aligned} \quad (\text{A-8})$$

Case 6—Pinned-Fixed:

$$\begin{aligned} \text{a. General } (P_1 > 0; P_2 > 0): \\ \text{SIN}(BZ)\{2 / PR - Z \times \text{SIN}(Z) \times [(1 + LR) / LR \\ + 1 / PR] - \text{COS}(Z) \times [PR / (1 + PR) + 2 / PR]\} \\ + \text{COS}(BZ)\{-B \times LR \times \text{SIN}(Z) + BZ \times \\ \text{COS}(Z)[1 + LR - 1 / (1 + PR)]\} = 0 \end{aligned} \quad (\text{A-16})$$

$$\begin{aligned} \text{b. } P_1 = 0: \\ \text{SIN}(BZ) \times \{(\text{BZ} \times \text{LR})^2 - 6\text{IR} / (\text{BZ} \times \text{LR})^2 - 6[1 + \\ (1 / \text{LR})]\} + 2\text{BZ} \times \text{COS}(BZ) \times \{[3\text{IR} / (\text{BZ} \times \text{LR})^2] \\ - \text{LR}\} = 0 \end{aligned} \quad (\text{A-17})$$

$$\begin{aligned} \text{c. } P_2 = 0: \\ B \times LR \times \text{COS}(BZ) \times [\text{SIN}(Z) - Z \times \text{COS}(Z) \times (1 + \\ LR) / LR] + \text{SIN}(BZ) \times [\text{COS}(Z) + Z \times \text{SIN}(Z) \times \\ (1 + LR) / LR] = 0 \end{aligned} \quad (\text{A-18})$$

Case 7—Pinned-Slider:

$$\begin{aligned} \text{a. General } (P_1 > 0; P_2 > 0): \\ [1 / (1 + PR)] \times Z \times \text{SIN}(Z) \times \text{SIN}(BZ) - LR \times BZ \times \\ \text{COS}(Z) \times \text{COS}(BZ) = 0 \end{aligned} \quad (\text{A-19})$$

$$\begin{aligned} \text{b. } P_1 = 0: \\ LR \times BZ \times \text{SIN}(BZ) - \text{IR} \times \text{COS}(BZ) = 0 \end{aligned} \quad (\text{A-20})$$

$$\begin{aligned} \text{c. } P_2 = 0: \\ Z \times \text{SIN}(Z) \times \text{SIN}(BZ) - BZ \times LR \times \text{COS}(Z) \times \\ \text{COS}(BZ) = 0 \end{aligned} \quad (\text{A-21})$$

Reference 4 outlines a computer program similar to the one described in Ref. 1. This program was developed to calculate the roots of the characteristic equations. The solution routine which serves to find the lowest root was modified to improve the speed of convergence to the root. Residual values were calculated by spacing points at equal intervals until a sign change in the residual was observed. At this point, instead of halving the incremental value of Z, a new value for Z was calculated by interpolating the two values of Z, which gave residuals of differing signs. The process was repeated retaining two values for Z, which produced the smallest residuals for further interpolation. The last step was repeated several times, producing a much faster convergence to the characteristic root.

The output from this program (Table 1) lists the slenderness ratios for all seven end-fixity types (Fig. 1) for a wide selection of segment inertia ratios, segment length ratios, and top-and-step-level axial-load ratios. Any intermediate value can be easily interpolated from the values presented.

Note that the axial load ratio $P_2 / P_T = P_2 / (P_1 + P_2)$ varies from 0 to 1. A value of zero corresponds to $P_1 > 0, P_2 = 0$ and a value of 1 corresponds to $P_1 = 0$ and $P_2 > 0$. All other values of the ratio correspond to P_1 and P_2 both greater than zero.

ACKNOWLEDGMENT

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Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_1(\text{eff}) / L_T; K_2 = l_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | |
|---------------------------------|-------|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|
| I1 | I2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | LT | PT | | | | | | | | | | | | | | |
| 0.1 | 0.1 | 0.0 | 0.997 | 3.153 | 1.820 | 5.755 | 0.637 | 2.014 | 0.910 | 2.878 | 0.455 | 1.439 | 0.695 | 2.198 | 1.999 | 6.320 |
| | | 0.2 | 1.020 | 2.886 | 1.820 | 5.148 | 0.637 | 1.802 | 0.910 | 2.574 | 0.455 | 1.287 | 0.719 | 2.033 | 2.047 | 5.791 |
| | | 0.4 | 1.059 | 2.595 | 1.820 | 4.458 | 0.637 | 1.560 | 0.910 | 2.229 | 0.455 | 1.115 | 0.759 | 1.858 | 2.127 | 5.211 |
| | | 0.6 | 1.137 | 2.275 | 1.820 | 3.640 | 0.637 | 1.274 | 0.910 | 1.820 | 0.455 | 0.910 | 0.836 | 1.672 | 2.282 | 4.565 |
| | | 0.8 | 1.360 | 1.923 | 1.820 | 2.574 | 0.637 | 0.901 | 0.910 | 1.287 | 0.455 | 0.644 | 1.045 | 1.478 | 2.708 | 3.829 |
| | | 1.0 | 0.000 | 1.560 | 0.000 | 0.200 | 0.000 | 0.197 | 0.000 | 0.199 | 0.000 | 0.196 | 0.000 | 1.283 | 0.000 | 2.997 |
| | | 0.1 | 0.3 | 0.0 | 0.941 | 2.975 | 1.461 | 4.621 | 0.523 | 1.654 | 0.732 | 2.314 | 0.370 | 1.170 | 0.629 | 1.989 |
| 0.2 | 0.982 | | | 2.779 | 1.461 | 4.133 | 0.524 | 1.483 | 0.732 | 2.071 | 0.372 | 1.051 | 0.664 | 1.879 | 2.088 | 5.906 |
| 0.4 | 1.050 | | | 2.572 | 1.461 | 3.580 | 0.526 | 1.289 | 0.733 | 1.795 | 0.375 | 0.918 | 0.720 | 1.764 | 2.283 | 5.593 |
| 0.6 | 1.176 | | | 2.353 | 1.462 | 2.924 | 0.531 | 1.061 | 0.735 | 1.469 | 0.384 | 0.769 | 0.822 | 1.644 | 2.633 | 5.267 |
| 0.8 | 1.500 | | | 2.121 | 1.464 | 2.071 | 0.552 | 0.780 | 0.742 | 1.049 | 0.439 | 0.621 | 1.074 | 1.519 | 3.487 | 4.932 |
| 1.0 | 0.000 | | | 1.880 | 0.000 | 0.600 | 0.000 | 0.556 | 0.000 | 0.590 | 0.000 | 0.531 | 0.000 | 1.389 | 0.000 | 4.631 |
| 0.1 | 0.5 | | | 0.0 | 0.793 | 2.506 | 1.107 | 3.502 | 0.468 | 1.480 | 0.566 | 1.788 | 0.342 | 1.081 | 0.493 | 1.561 |
| | | 0.2 | 0.826 | 2.336 | 1.109 | 3.135 | 0.479 | 1.356 | 0.572 | 1.616 | 0.356 | 1.006 | 0.518 | 1.464 | 2.008 | 5.679 |
| | | 0.4 | 0.880 | 2.156 | 1.112 | 2.724 | 0.499 | 1.223 | 0.583 | 1.429 | 0.379 | 0.928 | 0.556 | 1.361 | 2.255 | 5.524 |
| | | 0.6 | 0.980 | 1.960 | 1.120 | 2.239 | 0.540 | 1.080 | 0.614 | 1.229 | 0.422 | 0.845 | 0.626 | 1.252 | 2.691 | 5.381 |
| | | 0.8 | 1.234 | 1.746 | 1.148 | 1.623 | 0.659 | 0.932 | 0.747 | 1.056 | 0.535 | 0.757 | 0.801 | 1.133 | 3.696 | 5.227 |
| | | 1.0 | 0.000 | 1.513 | 0.000 | 1.000 | 0.000 | 0.786 | 0.000 | 0.962 | 0.000 | 0.665 | 0.000 | 1.004 | 0.000 | 5.074 |
| | | 0.1 | 0.7 | 0.0 | 0.561 | 1.775 | 0.785 | 2.482 | 0.436 | 1.378 | 0.482 | 1.524 | 0.279 | 0.883 | 0.331 | 1.045 |
| 0.2 | 0.579 | | | 1.638 | 0.800 | 2.263 | 0.448 | 1.268 | 0.518 | 1.467 | 0.289 | 0.816 | 0.345 | 0.975 | 1.755 | 4.963 |
| 0.4 | 0.608 | | | 1.489 | 0.828 | 2.029 | 0.469 | 1.148 | 0.577 | 1.414 | 0.303 | 0.743 | 0.369 | 0.903 | 2.006 | 4.914 |
| 0.6 | 0.662 | | | 1.325 | 0.893 | 1.786 | 0.508 | 1.016 | 0.683 | 1.367 | 0.331 | 0.663 | 0.417 | 0.833 | 2.429 | 4.858 |
| 0.8 | 0.807 | | | 1.141 | 1.103 | 1.560 | 0.612 | 0.865 | 0.937 | 1.325 | 0.405 | 0.573 | 0.546 | 0.772 | 3.395 | 4.802 |
| 1.0 | 0.000 | | | 0.939 | 0.000 | 1.400 | 0.000 | 0.689 | 0.000 | 1.289 | 0.000 | 0.469 | 0.000 | 0.729 | 0.000 | 4.734 |
| 0.1 | 0.9 | | | 0.0 | 0.329 | 1.039 | 0.638 | 2.016 | 0.245 | 0.774 | 0.461 | 1.456 | 0.194 | 0.614 | 0.275 | 0.869 |
| | | 0.2 | 0.356 | 1.007 | 0.697 | 1.971 | 0.257 | 0.727 | 0.512 | 1.448 | 0.214 | 0.607 | 0.305 | 0.863 | 1.219 | 3.446 |
| | | 0.4 | 0.399 | 0.977 | 0.787 | 1.927 | 0.279 | 0.683 | 0.588 | 1.441 | 0.245 | 0.600 | 0.350 | 0.858 | 1.405 | 3.441 |
| | | 0.6 | 0.476 | 0.951 | 0.942 | 1.884 | 0.324 | 0.648 | 0.716 | 1.433 | 0.297 | 0.594 | 0.426 | 0.853 | 1.718 | 3.437 |
| | | 0.8 | 0.657 | 0.929 | 1.302 | 1.841 | 0.440 | 0.623 | 1.008 | 1.425 | 0.417 | 0.589 | 0.599 | 0.848 | 2.425 | 3.430 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.606 | 0.000 | 1.418 | 0.000 | 0.000 | 0.000 | 0.843 | 0.000 | 3.417 |
| | | 0.2 | 0.1 | 0.0 | 0.998 | 2.230 | 1.840 | 4.115 | 0.645 | 1.441 | 0.920 | 2.057 | 0.460 | 1.029 | 0.696 | 1.555 |
| 0.2 | 1.021 | | | 2.042 | 1.840 | 3.680 | 0.645 | 1.289 | 0.920 | 1.840 | 0.460 | 0.921 | 0.720 | 1.439 | 2.047 | 4.095 |
| 0.4 | 1.060 | | | 1.836 | 1.840 | 3.187 | 0.645 | 1.117 | 0.920 | 1.594 | 0.461 | 0.798 | 0.760 | 1.316 | 2.128 | 3.685 |
| 0.6 | 1.139 | | | 1.610 | 1.840 | 2.602 | 0.645 | 0.912 | 0.920 | 1.302 | 0.461 | 0.652 | 0.838 | 1.185 | 2.283 | 3.229 |
| 0.8 | 1.363 | | | 1.363 | 1.840 | 1.840 | 0.646 | 0.646 | 0.921 | 0.921 | 0.462 | 0.462 | 1.049 | 1.049 | 2.710 | 2.710 |
| 1.0 | 0.000 | | | 1.109 | 0.000 | 0.200 | 0.000 | 0.194 | 0.000 | 0.198 | 0.000 | 0.192 | 0.000 | 0.914 | 0.000 | 2.123 |
| 0.2 | 0.3 | | | 0.0 | 0.947 | 2.117 | 1.523 | 3.406 | 0.557 | 1.246 | 0.765 | 1.711 | 0.396 | 0.887 | 0.635 | 1.421 |
| | | 0.2 | 0.990 | 1.979 | 1.523 | 3.047 | 0.561 | 1.121 | 0.767 | 1.534 | 0.403 | 0.805 | 0.672 | 1.344 | 2.093 | 4.187 |
| | | 0.4 | 1.059 | 1.835 | 1.525 | 2.641 | 0.567 | 0.983 | 0.771 | 1.335 | 0.415 | 0.719 | 0.730 | 1.265 | 2.290 | 3.966 |
| | | 0.6 | 1.190 | 1.683 | 1.528 | 2.160 | 0.584 | 0.826 | 0.778 | 1.100 | 0.446 | 0.630 | 0.836 | 1.183 | 2.646 | 3.743 |
| | | 0.8 | 1.523 | 1.523 | 1.537 | 1.537 | 0.653 | 0.653 | 0.812 | 0.812 | 0.549 | 0.549 | 1.097 | 1.097 | 3.508 | 3.508 |
| | | 1.0 | 0.000 | 1.358 | 0.000 | 0.600 | 0.000 | 0.522 | 0.000 | 0.581 | 0.000 | 0.484 | 0.000 | 1.010 | 0.000 | 3.297 |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = I_1(\text{eff}) / L_T; K_2 = I_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|
| I1 | L2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | -- | -- | | | | | | | | | | | | | | |
| I2 | LT | PT | | | | | | | | | | | | | | |
| 0.2 | 0.5 | 0.0 | 0.813 | 1.818 | 1.221 | 2.730 | 0.539 | 1.204 | 0.645 | 1.443 | 0.384 | 0.860 | 0.512 | 1.145 | 1.860 | 4.159 |
| | | 0.2 | 0.848 | 1.696 | 1.227 | 2.453 | 0.555 | 1.110 | 0.664 | 1.328 | 0.402 | 0.804 | 0.539 | 1.077 | 2.030 | 4.060 |
| | | 0.4 | 0.906 | 1.569 | 1.239 | 2.146 | 0.584 | 1.012 | 0.698 | 1.210 | 0.430 | 0.745 | 0.581 | 1.007 | 2.289 | 3.964 |
| | | 0.6 | 1.015 | 1.435 | 1.265 | 1.788 | 0.641 | 0.906 | 0.775 | 1.096 | 0.482 | 0.682 | 0.659 | 0.932 | 2.738 | 3.872 |
| | | 0.8 | 1.287 | 1.287 | 1.365 | 1.365 | 0.794 | 0.794 | 1.000 | 1.000 | 0.616 | 0.616 | 0.852 | 0.852 | 3.762 | 3.762 |
| | | 1.0 | 0.000 | 1.126 | 0.000 | 1.000 | 0.000 | 0.678 | 0.000 | 0.930 | 0.000 | 0.545 | 0.000 | 0.767 | 0.000 | 3.640 |
| 0.2 | 0.7 | 0.0 | 0.609 | 1.361 | 0.983 | 2.197 | 0.478 | 1.069 | 0.618 | 1.382 | 0.304 | 0.680 | 0.387 | 0.864 | 1.640 | 3.667 |
| | | 0.2 | 0.631 | 1.263 | 1.017 | 2.034 | 0.493 | 0.986 | 0.671 | 1.343 | 0.316 | 0.632 | 0.413 | 0.826 | 1.809 | 3.619 |
| | | 0.4 | 0.670 | 1.160 | 1.078 | 1.868 | 0.517 | 0.896 | 0.754 | 1.306 | 0.335 | 0.580 | 0.456 | 0.790 | 2.075 | 3.594 |
| | | 0.6 | 0.742 | 1.050 | 1.201 | 1.699 | 0.563 | 0.796 | 0.899 | 1.271 | 0.373 | 0.527 | 0.536 | 0.757 | 2.507 | 3.545 |
| | | 0.8 | 0.935 | 0.935 | 1.538 | 1.538 | 0.684 | 0.684 | 1.238 | 1.238 | 0.481 | 0.481 | 0.730 | 0.730 | 3.513 | 3.513 |
| | | 1.0 | 0.000 | 0.827 | 0.000 | 1.400 | 0.000 | 0.555 | 0.000 | 1.208 | 0.000 | 0.458 | 0.000 | 0.708 | 0.000 | 3.473 |
| 0.2 | 0.9 | 0.0 | 0.454 | 1.015 | 0.899 | 2.010 | 0.324 | 0.724 | 0.574 | 1.284 | 0.262 | 0.585 | 0.368 | 0.823 | 1.220 | 2.729 |
| | | 0.2 | 0.495 | 0.991 | 0.983 | 1.965 | 0.347 | 0.694 | 0.639 | 1.278 | 0.290 | 0.580 | 0.409 | 0.818 | 1.361 | 2.723 |
| | | 0.4 | 0.559 | 0.968 | 1.110 | 1.923 | 0.385 | 0.667 | 0.735 | 1.273 | 0.332 | 0.575 | 0.470 | 0.814 | 1.571 | 2.721 |
| | | 0.6 | 0.670 | 0.947 | 1.332 | 1.883 | 0.455 | 0.643 | 0.897 | 1.269 | 0.403 | 0.571 | 0.573 | 0.811 | 1.923 | 2.719 |
| | | 0.8 | 0.927 | 0.927 | 1.843 | 1.843 | 0.622 | 0.622 | 1.264 | 1.264 | 0.566 | 0.566 | 0.807 | 0.807 | 2.716 | 2.716 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.259 | 0.000 | 0.562 | 0.000 | 0.803 | 0.000 | 2.706 |
| 0.3 | 0.1 | 0.0 | 0.998 | 1.822 | 1.860 | 3.396 | 0.652 | 1.191 | 0.930 | 1.698 | 0.465 | 0.850 | 0.696 | 1.271 | 1.999 | 3.650 |
| | | 0.2 | 1.021 | 1.668 | 1.860 | 3.038 | 0.652 | 1.065 | 0.930 | 1.519 | 0.466 | 0.761 | 0.720 | 1.176 | 2.048 | 3.344 |
| | | 0.4 | 1.061 | 1.500 | 1.860 | 2.631 | 0.653 | 0.923 | 0.930 | 1.316 | 0.466 | 0.659 | 0.761 | 1.076 | 2.128 | 3.010 |
| | | 0.6 | 1.140 | 1.316 | 1.860 | 2.148 | 0.653 | 0.754 | 0.931 | 1.075 | 0.467 | 0.539 | 0.840 | 0.970 | 2.284 | 2.637 |
| | | 0.8 | 1.367 | 1.116 | 1.861 | 1.519 | 0.655 | 0.535 | 0.932 | 0.761 | 0.469 | 0.383 | 1.053 | 0.860 | 2.711 | 2.214 |
| | | 1.0 | 0.000 | 0.910 | 0.000 | 0.200 | 0.000 | 0.191 | 0.000 | 0.197 | 0.000 | 0.188 | 0.000 | 0.751 | 0.000 | 1.736 |
| 0.3 | 0.3 | 0.0 | 0.953 | 1.740 | 1.587 | 2.897 | 0.587 | 1.072 | 0.800 | 1.460 | 0.420 | 0.768 | 0.642 | 1.172 | 1.974 | 3.604 |
| | | 0.2 | 0.997 | 1.628 | 1.586 | 2.590 | 0.593 | 0.969 | 0.803 | 1.312 | 0.431 | 0.703 | 0.680 | 1.111 | 2.099 | 3.428 |
| | | 0.4 | 1.070 | 1.513 | 1.590 | 2.248 | 0.605 | 0.856 | 0.811 | 1.146 | 0.450 | 0.636 | 0.741 | 1.048 | 2.300 | 3.252 |
| | | 0.6 | 1.204 | 1.390 | 1.595 | 1.842 | 0.633 | 0.731 | 0.827 | 0.955 | 0.492 | 0.568 | 0.851 | 0.983 | 2.658 | 3.069 |
| | | 0.8 | 1.547 | 1.263 | 1.616 | 1.319 | 0.734 | 0.600 | 0.899 | 0.734 | 0.619 | 0.505 | 1.122 | 0.916 | 3.529 | 2.881 |
| | | 1.0 | 0.000 | 1.133 | 0.000 | 0.600 | 0.000 | 0.495 | 0.000 | 0.572 | 0.000 | 0.450 | 0.000 | 0.846 | 0.000 | 2.710 |
| 0.3 | 0.5 | 0.0 | 0.833 | 1.521 | 1.335 | 2.438 | 0.578 | 1.055 | 0.718 | 1.311 | 0.407 | 0.744 | 0.533 | 0.973 | 1.890 | 3.450 |
| | | 0.2 | 0.872 | 1.424 | 1.345 | 2.197 | 0.598 | 0.977 | 0.746 | 1.219 | 0.427 | 0.697 | 0.563 | 0.919 | 2.048 | 3.344 |
| | | 0.4 | 0.934 | 1.321 | 1.368 | 1.935 | 0.632 | 0.894 | 0.797 | 1.128 | 0.458 | 0.647 | 0.610 | 0.863 | 2.318 | 3.278 |
| | | 0.6 | 1.051 | 1.214 | 1.416 | 1.635 | 0.697 | 0.805 | 0.901 | 1.040 | 0.516 | 0.595 | 0.697 | 0.805 | 2.772 | 3.201 |
| | | 0.8 | 1.342 | 1.096 | 1.586 | 1.295 | 0.870 | 0.711 | 1.181 | 0.964 | 0.661 | 0.540 | 0.911 | 0.744 | 3.829 | 3.126 |
| | | 1.0 | 0.000 | 0.971 | 0.000 | 1.000 | 0.000 | 0.613 | 0.000 | 0.903 | 0.000 | 0.481 | 0.000 | 0.681 | 0.000 | 3.014 |
| 0.3 | 0.7 | 0.0 | 0.660 | 1.205 | 1.156 | 2.111 | 0.509 | 0.929 | 0.708 | 1.292 | 0.331 | 0.604 | 0.444 | 0.810 | 1.696 | 3.097 |
| | | 0.2 | 0.690 | 1.126 | 1.205 | 1.968 | 0.526 | 0.859 | 0.771 | 1.260 | 0.347 | 0.567 | 0.479 | 0.783 | 1.862 | 3.040 |
| | | 0.4 | 0.739 | 1.045 | 1.289 | 1.824 | 0.553 | 0.783 | 0.869 | 1.229 | 0.375 | 0.530 | 0.535 | 0.757 | 2.141 | 3.027 |
| | | 0.6 | 0.832 | 0.961 | 1.451 | 1.675 | 0.606 | 0.699 | 1.039 | 1.199 | 0.430 | 0.496 | 0.636 | 0.734 | 2.611 | 3.015 |
| | | 0.8 | 1.074 | 0.877 | 1.876 | 1.532 | 0.744 | 0.607 | 1.436 | 1.172 | 0.576 | 0.470 | 0.874 | 0.714 | 3.658 | 2.987 |
| | | 1.0 | 0.000 | 0.801 | 0.000 | 1.400 | 0.000 | 0.509 | 0.000 | 1.146 | 0.000 | 0.454 | 0.000 | 0.695 | 0.000 | 2.933 |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_{1(eff)} / L_T; K_2 = l_{2(eff)} / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|
| I1 | l2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | -- | -- | | | | | | | | | | | | | | |
| I2 | LT | PT | | | | | | | | | | | | | | |
| 0.3 | 0.9 | 0.0 | 0.553 | 1.009 | 1.101 | 2.010 | 0.390 | 0.712 | 0.651 | 1.189 | 0.309 | 0.564 | 0.433 | 0.790 | 1.342 | 2.450 |
| | | 0.2 | 0.604 | 0.986 | 1.202 | 1.963 | 0.421 | 0.687 | 0.726 | 1.186 | 0.343 | 0.560 | 0.482 | 0.787 | 1.495 | 2.441 |
| | | 0.4 | 0.683 | 0.966 | 1.362 | 1.926 | 0.469 | 0.663 | 0.836 | 1.183 | 0.393 | 0.556 | 0.554 | 0.784 | 1.729 | 2.445 |
| | | 0.6 | 0.819 | 0.946 | 1.632 | 1.884 | 0.556 | 0.641 | 1.021 | 1.179 | 0.479 | 0.553 | 0.677 | 0.781 | 2.115 | 2.442 |
| | | 0.8 | 1.135 | 0.927 | 2.255 | 1.841 | 0.762 | 0.622 | 1.441 | 1.176 | 0.673 | 0.549 | 0.953 | 0.779 | 2.988 | 2.440 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.172 | 0.000 | 0.546 | 0.000 | 0.775 | 0.000 | 2.430 |
| 0.4 | 0.1 | 0.0 | 0.998 | 1.578 | 1.880 | 2.973 | 0.660 | 1.043 | 0.940 | 1.487 | 0.471 | 0.744 | 0.696 | 1.101 | 1.999 | 3.161 |
| | | 0.2 | 1.022 | 1.445 | 1.880 | 2.659 | 0.660 | 0.933 | 0.941 | 1.330 | 0.471 | 0.666 | 0.721 | 1.020 | 2.048 | 2.896 |
| | | 0.4 | 1.061 | 1.300 | 1.880 | 2.303 | 0.660 | 0.809 | 0.941 | 1.152 | 0.472 | 0.578 | 0.762 | 0.933 | 2.128 | 2.607 |
| | | 0.6 | 1.141 | 1.141 | 1.881 | 1.881 | 0.661 | 0.661 | 0.941 | 0.941 | 0.473 | 0.473 | 0.842 | 0.842 | 2.285 | 2.285 |
| | | 0.8 | 1.370 | 0.969 | 1.882 | 1.330 | 0.664 | 0.469 | 0.943 | 0.667 | 0.478 | 0.338 | 1.058 | 0.748 | 2.713 | 1.919 |
| | | 1.0 | 0.000 | 0.792 | 0.000 | 0.200 | 0.000 | 0.189 | 0.000 | 0.197 | 0.000 | 0.185 | 0.000 | 0.656 | 0.000 | 1.506 |
| 0.4 | 0.3 | 0.0 | 0.959 | 1.517 | 1.648 | 2.606 | 0.612 | 0.968 | 0.833 | 1.318 | 0.440 | 0.695 | 0.649 | 1.027 | 1.980 | 3.131 |
| | | 0.2 | 1.005 | 1.421 | 1.649 | 2.332 | 0.621 | 0.878 | 0.840 | 1.187 | 0.453 | 0.640 | 0.689 | 0.975 | 2.105 | 2.977 |
| | | 0.4 | 1.079 | 1.322 | 1.655 | 2.027 | 0.637 | 0.780 | 0.851 | 1.042 | 0.476 | 0.583 | 0.752 | 0.922 | 2.308 | 2.826 |
| | | 0.6 | 1.218 | 1.218 | 1.665 | 1.665 | 0.674 | 0.674 | 0.878 | 0.878 | 0.526 | 0.526 | 0.867 | 0.867 | 2.666 | 2.666 |
| | | 0.8 | 1.571 | 1.111 | 1.698 | 1.201 | 0.798 | 0.564 | 0.985 | 0.696 | 0.668 | 0.472 | 1.146 | 0.810 | 3.549 | 2.509 |
| | | 1.0 | 0.000 | 1.002 | 0.000 | 0.600 | 0.000 | 0.472 | 0.000 | 0.563 | 0.000 | 0.424 | 0.000 | 0.753 | 0.000 | 2.362 |
| 0.4 | 0.5 | 0.0 | 0.857 | 1.355 | 1.451 | 2.294 | 0.605 | 0.957 | 0.779 | 1.232 | 0.424 | 0.670 | 0.556 | 0.879 | 1.908 | 3.017 |
| | | 0.2 | 0.897 | 1.268 | 1.460 | 2.065 | 0.628 | 0.888 | 0.815 | 1.153 | 0.445 | 0.629 | 0.589 | 0.833 | 2.076 | 2.936 |
| | | 0.4 | 0.964 | 1.181 | 1.494 | 1.830 | 0.665 | 0.815 | 0.878 | 1.075 | 0.478 | 0.586 | 0.642 | 0.787 | 2.352 | 2.881 |
| | | 0.6 | 1.088 | 1.088 | 1.560 | 1.560 | 0.737 | 0.737 | 1.002 | 1.002 | 0.541 | 0.541 | 0.738 | 0.738 | 2.829 | 2.829 |
| | | 0.8 | 1.401 | 0.991 | 1.789 | 1.265 | 0.925 | 0.654 | 1.320 | 0.933 | 0.697 | 0.493 | 0.974 | 0.688 | 3.903 | 2.760 |
| | | 1.0 | 0.000 | 0.885 | 0.000 | 1.000 | 0.000 | 0.568 | 0.000 | 0.877 | 0.000 | 0.442 | 0.000 | 0.639 | 0.000 | 2.650 |
| 0.4 | 0.7 | 0.0 | 0.713 | 1.127 | 1.309 | 2.069 | 0.537 | 0.849 | 0.775 | 1.225 | 0.359 | 0.568 | 0.494 | 0.781 | 1.751 | 2.768 |
| | | 0.2 | 0.749 | 1.060 | 1.370 | 1.938 | 0.556 | 0.787 | 0.845 | 1.195 | 0.380 | 0.538 | 0.537 | 0.759 | 1.919 | 2.714 |
| | | 0.4 | 0.809 | 0.990 | 1.475 | 1.806 | 0.588 | 0.720 | 0.954 | 1.169 | 0.416 | 0.510 | 0.603 | 0.738 | 2.212 | 2.709 |
| | | 0.6 | 0.921 | 0.921 | 1.670 | 1.670 | 0.648 | 0.648 | 1.143 | 1.143 | 0.484 | 0.484 | 0.719 | 0.719 | 2.700 | 2.700 |
| | | 0.8 | 1.206 | 0.853 | 2.164 | 1.530 | 0.807 | 0.570 | 1.584 | 1.120 | 0.657 | 0.464 | 0.991 | 0.701 | 3.785 | 2.677 |
| | | 1.0 | 0.000 | 0.791 | 0.000 | 1.400 | 0.000 | 0.494 | 0.000 | 1.097 | 0.000 | 0.450 | 0.000 | 0.685 | 0.000 | 2.622 |
| 0.4 | 0.9 | 0.0 | 0.637 | 1.006 | 1.272 | 2.011 | 0.447 | 0.707 | 0.715 | 1.130 | 0.347 | 0.548 | 0.484 | 0.766 | 1.460 | 2.309 |
| | | 0.2 | 0.696 | 0.984 | 1.389 | 1.964 | 0.484 | 0.684 | 0.798 | 1.128 | 0.385 | 0.545 | 0.540 | 0.764 | 1.618 | 2.289 |
| | | 0.4 | 0.788 | 0.965 | 1.570 | 1.922 | 0.540 | 0.662 | 0.920 | 1.127 | 0.442 | 0.542 | 0.622 | 0.761 | 1.877 | 2.298 |
| | | 0.6 | 0.946 | 0.946 | 1.891 | 1.891 | 0.641 | 0.641 | 1.124 | 1.124 | 0.539 | 0.539 | 0.760 | 0.760 | 2.305 | 2.305 |
| | | 0.8 | 1.312 | 0.928 | 2.613 | 1.847 | 0.880 | 0.622 | 1.586 | 1.122 | 0.758 | 0.536 | 1.071 | 0.758 | 3.257 | 2.303 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.118 | 0.000 | 0.533 | 0.000 | 0.754 | 0.000 | 2.282 |
| 0.5 | 0.1 | 0.0 | 0.998 | 1.412 | 1.901 | 2.688 | 0.667 | 0.943 | 0.951 | 1.344 | 0.476 | 0.673 | 0.697 | 0.985 | 2.000 | 2.828 |
| | | 0.2 | 1.022 | 1.293 | 1.900 | 2.404 | 0.667 | 0.844 | 0.951 | 1.202 | 0.477 | 0.603 | 0.722 | 0.913 | 2.048 | 2.591 |
| | | 0.4 | 1.062 | 1.164 | 1.901 | 2.082 | 0.668 | 0.731 | 0.951 | 1.042 | 0.477 | 0.523 | 0.763 | 0.836 | 2.129 | 2.332 |
| | | 0.6 | 1.143 | 1.022 | 1.901 | 1.700 | 0.669 | 0.598 | 0.952 | 0.851 | 0.479 | 0.429 | 0.844 | 0.755 | 2.285 | 2.044 |
| | | 0.8 | 1.373 | 0.868 | 1.902 | 1.203 | 0.673 | 0.426 | 0.955 | 0.604 | 0.487 | 0.308 | 1.062 | 0.672 | 2.715 | 1.717 |
| | | 1.0 | 0.000 | 0.712 | 0.000 | 0.200 | 0.000 | 0.186 | 0.000 | 0.196 | 0.000 | 0.192 | 0.000 | 0.590 | 0.000 | 1.349 |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_1(\text{eff}) / L_T; K_2 = l_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN - SLIDER | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|------------------|-------|
| I1 | L2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | -- | -- | | | | | | | | | | | | | | |
| I2 | LT | PT | | | | | | | | | | | | | | |
| 0.5 | 0.3 | 0.0 | 0.965 | 1.365 | 1.712 | 2.421 | 0.633 | 0.895 | 0.866 | 1.224 | 0.455 | 0.643 | 0.657 | 0.929 | 1.986 | 2.808 |
| | | 0.2 | 1.013 | 1.281 | 1.712 | 2.166 | 0.644 | 0.814 | 0.874 | 1.106 | 0.470 | 0.595 | 0.698 | 0.883 | 2.111 | 2.670 |
| | | 0.4 | 1.090 | 1.194 | 1.719 | 1.883 | 0.664 | 0.727 | 0.890 | 0.975 | 0.497 | 0.544 | 0.764 | 0.837 | 2.315 | 2.536 |
| | | 0.6 | 1.233 | 1.103 | 1.733 | 1.550 | 0.707 | 0.632 | 0.927 | 0.829 | 0.553 | 0.494 | 0.883 | 0.790 | 2.682 | 2.399 |
| | | 0.8 | 1.595 | 1.009 | 1.783 | 1.127 | 0.847 | 0.536 | 1.064 | 0.673 | 0.706 | 0.446 | 1.172 | 0.741 | 3.569 | 2.257 |
| | | 1.0 | 0.000 | 0.913 | 0.000 | 0.600 | 0.000 | 0.454 | 0.000 | 0.556 | 0.000 | 0.403 | 0.000 | 0.691 | 0.000 | 2.126 |
| 0.5 | 0.5 | 0.0 | 0.878 | 1.242 | 1.562 | 2.208 | 0.626 | 0.886 | 0.832 | 1.177 | 0.437 | 0.618 | 0.580 | 0.820 | 1.949 | 2.757 |
| | | 0.2 | 0.923 | 1.168 | 1.570 | 1.985 | 0.651 | 0.823 | 0.873 | 1.104 | 0.460 | 0.582 | 0.617 | 0.780 | 2.099 | 2.655 |
| | | 0.4 | 0.995 | 1.090 | 1.612 | 1.765 | 0.691 | 0.757 | 0.945 | 1.035 | 0.496 | 0.544 | 0.675 | 0.740 | 2.385 | 2.612 |
| | | 0.6 | 1.128 | 1.009 | 1.703 | 1.523 | 0.768 | 0.687 | 1.084 | 0.969 | 0.563 | 0.503 | 0.781 | 0.698 | 2.831 | 2.532 |
| | | 0.8 | 1.460 | 0.923 | 1.974 | 1.248 | 0.969 | 0.613 | 1.434 | 0.907 | 0.729 | 0.461 | 1.038 | 0.656 | 3.908 | 2.472 |
| | | 1.0 | 0.000 | 0.833 | 0.000 | 1.000 | 0.000 | 0.536 | 0.000 | 0.857 | 0.000 | 0.416 | 0.000 | 0.615 | 0.000 | 2.412 |
| 0.5 | 0.7 | 0.0 | 0.766 | 1.083 | 1.452 | 2.053 | 0.565 | 0.799 | 0.829 | 1.172 | 0.387 | 0.547 | 0.539 | 0.762 | 1.775 | 2.510 |
| | | 0.2 | 0.808 | 1.022 | 1.520 | 1.923 | 0.587 | 0.742 | 0.905 | 1.144 | 0.413 | 0.522 | 0.586 | 0.742 | 1.969 | 2.491 |
| | | 0.4 | 0.877 | 0.961 | 1.639 | 1.795 | 0.623 | 0.682 | 1.023 | 1.120 | 0.455 | 0.498 | 0.660 | 0.723 | 2.262 | 2.478 |
| | | 0.6 | 1.006 | 0.899 | 1.855 | 1.659 | 0.691 | 0.618 | 1.229 | 1.099 | 0.533 | 0.477 | 0.789 | 0.706 | 2.754 | 2.464 |
| | | 0.8 | 1.328 | 0.840 | 2.425 | 1.534 | 0.871 | 0.551 | 1.707 | 1.080 | 0.727 | 0.460 | 1.093 | 0.691 | 3.873 | 2.450 |
| | | 1.0 | 0.000 | 0.785 | 0.000 | 1.400 | 0.000 | 0.488 | 0.000 | 1.057 | 0.000 | 0.446 | 0.000 | 0.675 | 0.000 | 2.422 |
| 0.5 | 0.9 | 0.0 | 0.710 | 1.004 | 1.416 | 2.002 | 0.498 | 0.704 | 0.772 | 1.092 | 0.379 | 0.536 | 0.529 | 0.748 | 1.572 | 2.223 |
| | | 0.2 | 0.777 | 0.983 | 1.553 | 1.964 | 0.539 | 0.682 | 0.861 | 1.089 | 0.421 | 0.533 | 0.590 | 0.746 | 1.736 | 2.196 |
| | | 0.4 | 0.879 | 0.964 | 1.763 | 1.931 | 0.603 | 0.661 | 0.993 | 1.088 | 0.484 | 0.530 | 0.680 | 0.745 | 2.019 | 2.211 |
| | | 0.6 | 1.056 | 0.945 | 2.117 | 1.893 | 0.716 | 0.641 | 1.215 | 1.087 | 0.590 | 0.528 | 0.831 | 0.743 | 2.482 | 2.220 |
| | | 0.8 | 1.467 | 0.928 | 2.933 | 1.855 | 0.984 | 0.622 | 1.716 | 1.085 | 0.831 | 0.525 | 1.173 | 0.742 | 3.509 | 2.219 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.081 | 0.000 | 0.523 | 0.000 | 0.739 | 0.000 | 2.190 |
| 0.6 | 0.1 | 0.0 | 0.999 | 1.289 | 1.921 | 2.480 | 0.674 | 0.870 | 0.961 | 1.240 | 0.481 | 0.621 | 0.697 | 0.900 | 2.000 | 2.581 |
| | | 0.2 | 1.023 | 1.181 | 1.921 | 2.218 | 0.674 | 0.778 | 0.961 | 1.110 | 0.482 | 0.556 | 0.722 | 0.834 | 2.049 | 2.365 |
| | | 0.4 | 1.063 | 1.063 | 1.921 | 1.921 | 0.675 | 0.675 | 0.961 | 0.961 | 0.483 | 0.483 | 0.764 | 0.764 | 2.129 | 2.129 |
| | | 0.6 | 1.144 | 0.934 | 1.921 | 1.569 | 0.677 | 0.553 | 0.963 | 0.786 | 0.486 | 0.396 | 0.846 | 0.691 | 2.286 | 1.867 |
| | | 0.8 | 1.376 | 0.795 | 1.923 | 1.110 | 0.683 | 0.394 | 0.966 | 0.558 | 0.497 | 0.287 | 1.066 | 0.616 | 2.717 | 1.569 |
| | | 1.0 | 0.000 | 0.854 | 0.000 | 0.200 | 0.000 | 0.184 | 0.000 | 0.195 | 0.000 | 0.179 | 0.000 | 0.543 | 0.000 | 1.234 |
| 0.6 | 0.3 | 0.0 | 0.973 | 1.256 | 1.774 | 2.290 | 0.651 | 0.840 | 0.897 | 1.158 | 0.467 | 0.603 | 0.665 | 0.858 | 1.992 | 2.572 |
| | | 0.2 | 1.021 | 1.179 | 1.774 | 2.048 | 0.663 | 0.766 | 0.907 | 1.048 | 0.484 | 0.559 | 0.708 | 0.817 | 2.115 | 2.442 |
| | | 0.4 | 1.100 | 1.100 | 1.785 | 1.785 | 0.686 | 0.686 | 0.927 | 0.927 | 0.514 | 0.514 | 0.776 | 0.776 | 2.323 | 2.323 |
| | | 0.6 | 1.248 | 1.019 | 1.803 | 1.472 | 0.735 | 0.600 | 0.973 | 0.794 | 0.574 | 0.468 | 0.899 | 0.734 | 2.691 | 2.197 |
| | | 0.8 | 1.620 | 0.935 | 1.867 | 1.078 | 0.889 | 0.513 | 1.135 | 0.655 | 0.737 | 0.425 | 1.197 | 0.691 | 3.590 | 2.073 |
| | | 1.0 | 0.000 | 0.850 | 0.000 | 0.600 | 0.000 | 0.438 | 0.000 | 0.549 | 0.000 | 0.386 | 0.000 | 0.648 | 0.000 | 1.953 |
| 0.6 | 0.5 | 0.0 | 0.903 | 1.166 | 1.654 | 2.135 | 0.644 | 0.831 | 0.877 | 1.132 | 0.450 | 0.581 | 0.604 | 0.780 | 1.967 | 2.539 |
| | | 0.2 | 0.950 | 1.097 | 1.678 | 1.937 | 0.670 | 0.774 | 0.922 | 1.064 | 0.475 | 0.548 | 0.645 | 0.745 | 2.118 | 2.445 |
| | | 0.4 | 1.027 | 1.027 | 1.726 | 1.726 | 0.713 | 0.713 | 1.001 | 1.001 | 0.513 | 0.513 | 0.709 | 0.709 | 2.411 | 2.411 |
| | | 0.6 | 1.167 | 0.953 | 1.837 | 1.500 | 0.795 | 0.649 | 1.153 | 0.941 | 0.584 | 0.476 | 0.822 | 0.671 | 2.864 | 2.338 |
| | | 0.8 | 1.522 | 0.879 | 2.151 | 1.242 | 1.007 | 0.581 | 1.531 | 0.884 | 0.759 | 0.438 | 1.100 | 0.635 | 3.958 | 2.285 |
| | | 1.0 | 0.000 | 0.796 | 0.000 | 1.000 | 0.000 | 0.512 | 0.000 | 0.836 | 0.000 | 0.398 | 0.000 | 0.599 | 0.000 | 2.238 |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_1(\text{eff}) / L_T; K_2 = l_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|
| I1 | L2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | -- | -- | | | | | | | | | | | | | | |
| I2 | LT | PT | | | | | | | | | | | | | | |
| 0.6 | 0.7 | 0.0 | 0.817 | 1.055 | 1.578 | 2.037 | 0.592 | 0.765 | 0.873 | 1.128 | 0.413 | 0.534 | 0.578 | 0.746 | 1.860 | 2.401 |
| | | 0.2 | 0.864 | 0.998 | 1.656 | 1.913 | 0.617 | 0.712 | 0.954 | 1.101 | 0.443 | 0.511 | 0.630 | 0.728 | 2.026 | 2.339 |
| | | 0.4 | 0.942 | 0.942 | 1.793 | 1.793 | 0.657 | 0.657 | 1.083 | 1.083 | 0.490 | 0.490 | 0.711 | 0.711 | 2.350 | 2.350 |
| | | 0.6 | 1.087 | 0.887 | 2.027 | 1.656 | 0.734 | 0.599 | 1.302 | 1.063 | 0.577 | 0.471 | 0.851 | 0.695 | 2.887 | 2.358 |
| | | 0.8 | 1.441 | 0.832 | 2.660 | 1.536 | 0.935 | 0.540 | 1.804 | 1.041 | 0.789 | 0.455 | 1.180 | 0.682 | 4.058 | 2.343 |
| | | 1.0 | 0.000 | 0.782 | 0.000 | 1.400 | 0.000 | 0.486 | 0.000 | 1.024 | 0.000 | 0.442 | 0.000 | 0.667 | 0.000 | 2.278 |
| 0.6 | 0.9 | 0.0 | 0.776 | 1.002 | 1.560 | 2.014 | 0.544 | 0.702 | 0.825 | 1.065 | 0.407 | 0.525 | 0.568 | 0.734 | 1.663 | 2.147 |
| | | 0.2 | 0.851 | 0.982 | 1.704 | 1.968 | 0.590 | 0.681 | 0.919 | 1.061 | 0.453 | 0.523 | 0.634 | 0.733 | 1.850 | 2.136 |
| | | 0.4 | 0.963 | 0.963 | 1.934 | 1.934 | 0.660 | 0.660 | 1.062 | 1.062 | 0.521 | 0.521 | 0.731 | 0.731 | 2.148 | 2.148 |
| | | 0.6 | 1.156 | 0.944 | 2.330 | 1.903 | 0.784 | 0.640 | 1.298 | 1.060 | 0.636 | 0.519 | 0.895 | 0.731 | 2.626 | 2.144 |
| | | 0.8 | 1.605 | 0.926 | 3.227 | 1.863 | 1.077 | 0.622 | 1.833 | 1.059 | 0.896 | 0.517 | 1.263 | 0.729 | 3.711 | 2.143 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.055 | 0.000 | 0.515 | 0.000 | 0.726 | 0.000 | 2.127 |
| 0.7 | 0.1 | 0.0 | 0.999 | 1.194 | 1.941 | 2.320 | 0.680 | 0.813 | 0.971 | 1.160 | 0.486 | 0.581 | 0.698 | 0.834 | 2.000 | 2.390 |
| | | 0.2 | 1.023 | 1.094 | 1.941 | 2.075 | 0.681 | 0.728 | 0.971 | 1.038 | 0.487 | 0.521 | 0.723 | 0.773 | 2.049 | 2.190 |
| | | 0.4 | 1.064 | 0.985 | 1.941 | 1.797 | 0.682 | 0.632 | 0.972 | 0.900 | 0.488 | 0.452 | 0.765 | 0.709 | 2.130 | 1.972 |
| | | 0.6 | 1.145 | 0.866 | 1.942 | 1.468 | 0.684 | 0.517 | 0.973 | 0.736 | 0.492 | 0.372 | 0.848 | 0.641 | 2.287 | 1.729 |
| | | 0.8 | 1.380 | 0.737 | 1.944 | 1.039 | 0.692 | 0.370 | 0.978 | 0.523 | 0.507 | 0.271 | 1.071 | 0.572 | 2.719 | 1.453 |
| | | 1.0 | 0.000 | 0.608 | 0.000 | 0.200 | 0.000 | 0.182 | 0.000 | 0.194 | 0.000 | 0.176 | 0.000 | 0.506 | 0.000 | 1.145 |
| 0.7 | 0.3 | 0.0 | 0.979 | 1.171 | 1.835 | 2.194 | 0.665 | 0.795 | 0.925 | 1.106 | 0.477 | 0.570 | 0.673 | 0.804 | 1.988 | 2.376 |
| | | 0.2 | 1.029 | 1.100 | 1.834 | 1.961 | 0.679 | 0.726 | 0.938 | 1.003 | 0.496 | 0.530 | 0.718 | 0.767 | 2.122 | 2.268 |
| | | 0.4 | 1.112 | 1.029 | 1.847 | 1.709 | 0.704 | 0.652 | 0.962 | 0.891 | 0.528 | 0.488 | 0.789 | 0.730 | 2.333 | 2.160 |
| | | 0.6 | 1.264 | 0.956 | 1.872 | 1.415 | 0.758 | 0.573 | 1.016 | 0.768 | 0.592 | 0.447 | 0.916 | 0.692 | 2.703 | 2.043 |
| | | 0.8 | 1.644 | 0.879 | 1.951 | 1.043 | 0.924 | 0.494 | 1.199 | 0.641 | 0.763 | 0.408 | 1.224 | 0.654 | 3.611 | 1.930 |
| | | 1.0 | 0.000 | 0.803 | 0.000 | 0.600 | 0.000 | 0.424 | 0.000 | 0.541 | 0.000 | 0.371 | 0.000 | 0.615 | 0.000 | 1.820 |
| 0.7 | 0.5 | 0.0 | 0.926 | 1.107 | 1.743 | 2.083 | 0.659 | 0.788 | 0.911 | 1.089 | 0.463 | 0.553 | 0.629 | 0.752 | 1.956 | 2.338 |
| | | 0.2 | 0.977 | 1.045 | 1.773 | 1.895 | 0.687 | 0.734 | 0.964 | 1.030 | 0.488 | 0.522 | 0.673 | 0.719 | 2.142 | 2.290 |
| | | 0.4 | 1.059 | 0.980 | 1.836 | 1.700 | 0.733 | 0.678 | 1.049 | 0.971 | 0.529 | 0.490 | 0.741 | 0.686 | 2.428 | 2.248 |
| | | 0.6 | 1.208 | 0.913 | 1.958 | 1.480 | 0.819 | 0.619 | 1.210 | 0.915 | 0.604 | 0.457 | 0.864 | 0.653 | 2.901 | 2.193 |
| | | 0.8 | 1.584 | 0.846 | 2.312 | 1.236 | 1.042 | 0.557 | 1.615 | 0.863 | 0.789 | 0.422 | 1.159 | 0.620 | 4.018 | 2.148 |
| | | 1.0 | 0.000 | 0.772 | 0.000 | 1.000 | 0.000 | 0.493 | 0.000 | 0.820 | 0.000 | 0.386 | 0.000 | 0.587 | 0.000 | 2.105 |
| 0.7 | 0.7 | 0.0 | 0.866 | 1.036 | 1.692 | 2.023 | 0.620 | 0.741 | 0.913 | 1.091 | 0.438 | 0.523 | 0.612 | 0.732 | 1.890 | 2.259 |
| | | 0.2 | 0.918 | 0.982 | 1.785 | 1.908 | 0.647 | 0.692 | 0.998 | 1.066 | 0.470 | 0.502 | 0.669 | 0.715 | 2.086 | 2.230 |
| | | 0.4 | 1.004 | 0.929 | 1.926 | 1.784 | 0.692 | 0.641 | 1.134 | 1.050 | 0.522 | 0.483 | 0.756 | 0.700 | 2.387 | 2.210 |
| | | 0.6 | 1.161 | 0.878 | 2.205 | 1.667 | 0.776 | 0.587 | 1.363 | 1.030 | 0.617 | 0.466 | 0.907 | 0.685 | 2.949 | 2.229 |
| | | 0.8 | 1.550 | 0.829 | 2.880 | 1.539 | 0.997 | 0.533 | 1.895 | 1.013 | 0.845 | 0.451 | 1.257 | 0.672 | 4.153 | 2.220 |
| | | 1.0 | 0.000 | 0.780 | 0.000 | 1.400 | 0.000 | 0.484 | 0.000 | 0.996 | 0.000 | 0.439 | 0.000 | 0.660 | 0.000 | 2.169 |
| 0.7 | 0.9 | 0.0 | 0.839 | 1.003 | 1.677 | 2.005 | 0.587 | 0.701 | 0.873 | 1.044 | 0.433 | 0.517 | 0.604 | 0.723 | 1.779 | 2.127 |
| | | 0.2 | 0.918 | 0.982 | 1.840 | 1.968 | 0.636 | 0.680 | 0.973 | 1.040 | 0.482 | 0.515 | 0.675 | 0.721 | 1.955 | 2.090 |
| | | 0.4 | 1.040 | 0.963 | 2.100 | 1.944 | 0.712 | 0.660 | 1.125 | 1.042 | 0.555 | 0.513 | 0.778 | 0.720 | 2.267 | 2.099 |
| | | 0.6 | 1.249 | 0.944 | 2.523 | 1.907 | 0.847 | 0.640 | 1.376 | 1.040 | 0.677 | 0.512 | 0.951 | 0.719 | 2.812 | 2.126 |
| | | 0.8 | 1.740 | 0.930 | 3.475 | 1.857 | 1.164 | 0.622 | 1.944 | 1.039 | 0.954 | 0.510 | 1.343 | 0.718 | 3.976 | 2.125 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.035 | 0.000 | 0.509 | 0.000 | 0.716 | 0.000 | 2.081 |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_1(\text{eff}) / L_T; K_2 = l_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|-------|
| I1 | I2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | |
| -- | -- | -- | | | | | | | | | | | | | | | |
| | I2 | LT | PT | | | | | | | | | | | | | | |
| 0.8 | 0.1 | 0.0 | 0.0 | 0.999 | 1.117 | 1.961 | 2.192 | 0.687 | 0.768 | 0.980 | 1.096 | 0.491 | 0.549 | 0.698 | 0.781 | 2.000 | 2.236 |
| | | 0.2 | 1.023 | 1.023 | 1.961 | 1.961 | 0.688 | 0.688 | 0.981 | 0.981 | 0.492 | 0.492 | 0.724 | 0.724 | 2.049 | 2.049 | |
| | | 0.4 | 1.064 | 0.922 | 1.961 | 1.699 | 0.689 | 0.597 | 0.982 | 0.850 | 0.494 | 0.428 | 0.766 | 0.664 | 2.130 | 1.845 | |
| | | 0.6 | 1.147 | 0.811 | 1.962 | 1.388 | 0.692 | 0.489 | 0.984 | 0.696 | 0.498 | 0.352 | 0.850 | 0.601 | 2.288 | 1.618 | |
| | | 0.8 | 1.383 | 0.692 | 1.965 | 0.982 | 0.702 | 0.351 | 0.990 | 0.495 | 0.518 | 0.259 | 1.075 | 0.538 | 2.721 | 1.360 | |
| | | 1.0 | 0.000 | 0.572 | 0.000 | 0.200 | 0.000 | 0.180 | 0.000 | 0.193 | 0.000 | 0.173 | 0.000 | 0.477 | 0.000 | 1.072 | |
| 0.8 | 0.3 | 0.0 | 0.987 | 1.103 | 1.890 | 2.113 | 0.678 | 0.758 | 0.953 | 1.065 | 0.486 | 0.543 | 0.681 | 0.762 | 2.004 | 2.240 | |
| | | 0.2 | 1.038 | 1.038 | 1.896 | 1.896 | 0.693 | 0.693 | 0.967 | 0.967 | 0.506 | 0.506 | 0.728 | 0.728 | 2.128 | 2.128 | |
| | | 0.4 | 1.122 | 0.972 | 1.911 | 1.655 | 0.721 | 0.624 | 0.995 | 0.862 | 0.540 | 0.467 | 0.802 | 0.694 | 2.338 | 2.025 | |
| | | 0.6 | 1.279 | 0.904 | 1.940 | 1.372 | 0.779 | 0.551 | 1.056 | 0.747 | 0.607 | 0.429 | 0.933 | 0.660 | 2.721 | 1.924 | |
| | | 0.8 | 1.671 | 0.835 | 2.035 | 1.018 | 0.954 | 0.477 | 1.258 | 0.629 | 0.786 | 0.393 | 1.250 | 0.625 | 3.619 | 1.810 | |
| | | 1.0 | 0.000 | 0.765 | 0.000 | 0.600 | 0.000 | 0.412 | 0.000 | 0.536 | 0.000 | 0.359 | 0.000 | 0.589 | 0.000 | 1.713 | |
| 0.8 | 0.5 | 0.0 | 0.950 | 1.062 | 1.836 | 2.053 | 0.673 | 0.752 | 0.947 | 1.059 | 0.475 | 0.531 | 0.653 | 0.730 | 1.966 | 2.199 | |
| | | 0.2 | 1.005 | 1.005 | 1.868 | 1.868 | 0.703 | 0.703 | 1.002 | 1.002 | 0.502 | 0.502 | 0.700 | 0.700 | 2.165 | 2.165 | |
| | | 0.4 | 1.092 | 0.945 | 1.933 | 1.674 | 0.751 | 0.650 | 1.092 | 0.946 | 0.546 | 0.473 | 0.773 | 0.669 | 2.442 | 2.115 | |
| | | 0.6 | 1.250 | 0.884 | 2.078 | 1.469 | 0.842 | 0.595 | 1.262 | 0.893 | 0.624 | 0.442 | 0.903 | 0.639 | 2.930 | 2.072 | |
| | | 0.8 | 1.645 | 0.822 | 2.457 | 1.228 | 1.075 | 0.538 | 1.689 | 0.845 | 0.819 | 0.410 | 1.219 | 0.609 | 4.083 | 2.042 | |
| | | 1.0 | 0.000 | 0.753 | 0.000 | 1.000 | 0.000 | 0.478 | 0.000 | 0.803 | 0.000 | 0.376 | 0.000 | 0.579 | 0.000 | 2.000 | |
| 0.8 | 0.7 | 0.0 | 0.913 | 1.021 | 1.811 | 2.025 | 0.647 | 0.723 | 0.943 | 1.054 | 0.460 | 0.514 | 0.644 | 0.720 | 1.936 | 2.164 | |
| | | 0.2 | 0.970 | 0.970 | 1.895 | 1.895 | 0.677 | 0.677 | 1.035 | 1.035 | 0.495 | 0.495 | 0.705 | 0.705 | 2.133 | 2.133 | |
| | | 0.4 | 1.063 | 0.920 | 2.067 | 1.790 | 0.726 | 0.628 | 1.181 | 1.023 | 0.551 | 0.477 | 0.797 | 0.691 | 2.494 | 2.160 | |
| | | 0.6 | 1.234 | 0.872 | 2.364 | 1.671 | 0.818 | 0.578 | 1.424 | 1.007 | 0.652 | 0.461 | 0.957 | 0.677 | 2.992 | 2.115 | |
| | | 0.8 | 1.651 | 0.826 | 3.058 | 1.529 | 1.056 | 0.528 | 1.975 | 0.988 | 0.895 | 0.447 | 1.329 | 0.664 | 4.198 | 2.099 | |
| | | 1.0 | 0.000 | 0.778 | 0.000 | 1.400 | 0.000 | 0.483 | 0.000 | 0.973 | 0.000 | 0.436 | 0.000 | 0.653 | 0.000 | 2.084 | |
| 0.8 | 0.9 | 0.0 | 0.895 | 1.001 | 1.804 | 2.017 | 0.626 | 0.700 | 0.917 | 1.026 | 0.456 | 0.510 | 0.638 | 0.713 | 1.839 | 2.056 | |
| | | 0.2 | 0.981 | 0.981 | 1.971 | 1.971 | 0.679 | 0.679 | 1.024 | 1.024 | 0.509 | 0.509 | 0.713 | 0.713 | 2.053 | 2.053 | |
| | | 0.4 | 1.111 | 0.962 | 2.220 | 1.923 | 0.761 | 0.659 | 1.186 | 1.027 | 0.586 | 0.507 | 0.821 | 0.711 | 2.406 | 2.084 | |
| | | 0.6 | 1.335 | 0.944 | 2.705 | 1.913 | 0.905 | 0.640 | 1.447 | 1.023 | 0.716 | 0.506 | 1.004 | 0.710 | 2.907 | 2.056 | |
| | | 0.8 | 1.862 | 0.931 | 3.751 | 1.876 | 1.244 | 0.622 | 2.045 | 1.023 | 1.010 | 0.505 | 1.422 | 0.711 | 4.110 | 2.055 | |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.019 | 0.000 | 0.503 | 0.000 | 0.708 | 0.000 | 2.047 | |
| 0.9 | 0.1 | 0.0 | 1.000 | 1.054 | 1.980 | 2.087 | 0.693 | 0.731 | 0.990 | 1.044 | 0.495 | 0.522 | 0.699 | 0.737 | 2.001 | 2.109 | |
| | | 0.2 | 1.024 | 0.965 | 1.981 | 1.867 | 0.694 | 0.654 | 0.991 | 0.934 | 0.497 | 0.468 | 0.724 | 0.683 | 2.049 | 1.932 | |
| | | 0.4 | 1.065 | 0.870 | 1.981 | 1.618 | 0.696 | 0.568 | 0.992 | 0.810 | 0.499 | 0.408 | 0.767 | 0.627 | 2.131 | 1.740 | |
| | | 0.6 | 1.148 | 0.766 | 1.982 | 1.322 | 0.699 | 0.466 | 0.994 | 0.663 | 0.505 | 0.336 | 0.852 | 0.568 | 2.289 | 1.526 | |
| | | 0.8 | 1.386 | 0.654 | 1.986 | 0.936 | 0.711 | 0.335 | 1.003 | 0.473 | 0.528 | 0.249 | 1.080 | 0.509 | 2.722 | 1.283 | |
| | | 1.0 | 0.000 | 0.542 | 0.000 | 0.200 | 0.000 | 0.178 | 0.000 | 0.193 | 0.000 | 0.171 | 0.000 | 0.452 | 0.000 | 1.013 | |
| 0.9 | 0.3 | 0.0 | 0.994 | 1.048 | 1.954 | 2.059 | 0.689 | 0.727 | 0.978 | 1.031 | 0.493 | 0.520 | 0.690 | 0.728 | 2.002 | 2.110 | |
| | | 0.2 | 1.047 | 0.987 | 1.954 | 1.842 | 0.706 | 0.665 | 0.994 | 0.937 | 0.514 | 0.485 | 0.739 | 0.697 | 2.132 | 2.010 | |
| | | 0.4 | 1.134 | 0.926 | 1.971 | 1.610 | 0.735 | 0.600 | 1.025 | 0.837 | 0.550 | 0.449 | 0.815 | 0.665 | 2.348 | 1.918 | |
| | | 0.6 | 1.294 | 0.863 | 2.007 | 1.338 | 0.797 | 0.531 | 1.093 | 0.729 | 0.621 | 0.414 | 0.951 | 0.634 | 2.726 | 1.817 | |
| | | 0.8 | 1.696 | 0.799 | 2.115 | 0.997 | 0.981 | 0.463 | 1.311 | 0.618 | 0.807 | 0.380 | 1.276 | 0.602 | 3.654 | 1.722 | |
| | | 1.0 | 0.000 | 0.735 | 0.000 | 0.600 | 0.000 | 0.402 | 0.000 | 0.529 | 0.000 | 0.349 | 0.000 | 0.569 | 0.000 | 1.625 | |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_1(\text{eff}) / L_T; K_2 = l_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|
| I1 | L2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | -- | -- | | | | | | | | | | | | | | |
| I2 | LT | PT | | | | | | | | | | | | | | |
| 0.9 | 0.5 | 0.0 | 0.976 | 1.028 | 1.922 | 2.026 | 0.686 | 0.724 | 0.974 | 1.027 | 0.488 | 0.514 | 0.677 | 0.713 | 1.991 | 2.098 |
| | | 0.2 | 1.033 | 0.974 | 1.959 | 1.847 | 0.718 | 0.677 | 1.035 | 0.975 | 0.516 | 0.487 | 0.727 | 0.685 | 2.193 | 2.067 |
| | | 0.4 | 1.125 | 0.918 | 2.037 | 1.663 | 0.769 | 0.627 | 1.130 | 0.922 | 0.562 | 0.459 | 0.804 | 0.656 | 2.471 | 2.018 |
| | | 0.6 | 1.290 | 0.860 | 2.196 | 1.464 | 0.864 | 0.576 | 1.310 | 0.873 | 0.645 | 0.430 | 0.941 | 0.627 | 2.965 | 1.977 |
| | | 0.8 | 1.699 | 0.801 | 2.618 | 1.234 | 1.107 | 0.522 | 1.756 | 0.828 | 0.849 | 0.400 | 1.271 | 0.599 | 4.110 | 1.938 |
| | | 1.0 | 0.000 | 0.738 | 0.000 | 1.000 | 0.000 | 0.466 | 0.000 | 0.788 | 0.000 | 0.370 | 0.000 | 0.572 | 0.000 | 1.913 |
| 0.9 | 0.7 | 0.0 | 0.957 | 1.009 | 1.906 | 2.009 | 0.673 | 0.710 | 0.972 | 1.025 | 0.481 | 0.507 | 0.673 | 0.709 | 1.963 | 2.069 |
| | | 0.2 | 1.020 | 0.961 | 2.012 | 1.897 | 0.706 | 0.665 | 1.069 | 1.008 | 0.518 | 0.489 | 0.737 | 0.695 | 2.194 | 2.069 |
| | | 0.4 | 1.119 | 0.914 | 2.193 | 1.790 | 0.759 | 0.619 | 1.221 | 0.997 | 0.578 | 0.472 | 0.835 | 0.681 | 2.549 | 2.081 |
| | | 0.6 | 1.300 | 0.867 | 2.514 | 1.676 | 0.858 | 0.572 | 1.469 | 0.979 | 0.686 | 0.457 | 1.003 | 0.669 | 3.060 | 2.040 |
| | | 0.8 | 1.747 | 0.823 | 3.290 | 1.551 | 1.113 | 0.525 | 2.048 | 0.966 | 0.943 | 0.445 | 1.394 | 0.657 | 4.308 | 2.031 |
| | | 1.0 | 0.000 | 0.777 | 0.000 | 1.400 | 0.000 | 0.482 | 0.000 | 0.953 | 0.000 | 0.433 | 0.000 | 0.646 | 0.000 | 2.015 |
| 0.9 | 0.9 | 0.0 | 0.950 | 1.001 | 1.902 | 2.005 | 0.664 | 0.700 | 0.967 | 1.019 | 0.479 | 0.505 | 0.669 | 0.706 | 1.983 | 2.091 |
| | | 0.2 | 1.041 | 0.981 | 2.095 | 1.975 | 0.720 | 0.679 | 1.071 | 1.010 | 0.534 | 0.503 | 0.748 | 0.705 | 2.162 | 2.039 |
| | | 0.4 | 1.179 | 0.962 | 2.401 | 1.960 | 0.807 | 0.659 | 1.242 | 1.015 | 0.615 | 0.502 | 0.862 | 0.704 | 2.522 | 2.059 |
| | | 0.6 | 1.416 | 0.944 | 2.843 | 1.895 | 0.960 | 0.640 | 1.526 | 1.017 | 0.751 | 0.501 | 1.056 | 0.704 | 3.135 | 2.090 |
| | | 0.8 | 1.965 | 0.926 | 3.909 | 1.843 | 1.319 | 0.622 | 2.157 | 1.017 | 1.060 | 0.500 | 1.492 | 0.703 | 4.433 | 2.090 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 1.007 | 0.000 | 0.498 | 0.000 | 0.701 | 0.000 | 2.020 |
| 1.0 | 0.1 | 0.0 | 1.000 | 1.000 | 2.001 | 2.001 | 0.699 | 0.699 | 1.000 | 1.000 | 0.500 | 0.500 | 0.699 | 0.699 | 2.001 | 2.001 |
| | | 0.2 | 1.024 | 0.916 | 2.001 | 1.789 | 0.700 | 0.626 | 1.001 | 0.895 | 0.502 | 0.449 | 0.725 | 0.649 | 2.049 | 1.833 |
| | | 0.4 | 1.066 | 0.826 | 2.001 | 1.550 | 0.702 | 0.544 | 1.002 | 0.776 | 0.504 | 0.391 | 0.769 | 0.595 | 2.131 | 1.651 |
| | | 0.6 | 1.150 | 0.727 | 2.003 | 1.267 | 0.706 | 0.447 | 1.005 | 0.636 | 0.511 | 0.323 | 0.854 | 0.540 | 2.289 | 1.448 |
| | | 0.8 | 1.390 | 0.622 | 2.007 | 0.897 | 0.720 | 0.322 | 1.015 | 0.454 | 0.538 | 0.241 | 1.084 | 0.485 | 2.724 | 1.218 |
| | | 1.0 | 0.000 | 0.517 | 0.000 | 0.200 | 0.000 | 0.176 | 0.000 | 0.192 | 0.000 | 0.169 | 0.000 | 0.432 | 0.000 | 0.962 |
| 1.0 | 0.3 | 0.0 | 1.002 | 1.002 | 2.010 | 2.010 | 0.699 | 0.699 | 1.002 | 1.002 | 0.500 | 0.500 | 0.700 | 0.700 | 2.010 | 2.010 |
| | | 0.2 | 1.056 | 0.944 | 2.011 | 1.799 | 0.717 | 0.641 | 1.020 | 0.912 | 0.522 | 0.467 | 0.750 | 0.671 | 2.138 | 1.913 |
| | | 0.4 | 1.145 | 0.887 | 2.031 | 1.573 | 0.748 | 0.579 | 1.054 | 0.816 | 0.560 | 0.434 | 0.828 | 0.642 | 2.354 | 1.824 |
| | | 0.6 | 1.310 | 0.828 | 2.069 | 1.309 | 0.813 | 0.514 | 1.127 | 0.713 | 0.634 | 0.401 | 0.967 | 0.612 | 2.744 | 1.735 |
| | | 0.8 | 1.721 | 0.770 | 2.196 | 0.982 | 1.005 | 0.450 | 1.360 | 0.608 | 0.826 | 0.370 | 1.301 | 0.582 | 3.662 | 1.638 |
| | | 1.0 | 0.000 | 0.711 | 0.000 | 0.600 | 0.000 | 0.393 | 0.000 | 0.522 | 0.000 | 0.340 | 0.000 | 0.552 | 0.000 | 1.551 |
| 1.0 | 0.5 | 0.0 | 1.000 | 1.000 | 2.013 | 2.013 | 0.699 | 0.699 | 1.000 | 1.000 | 0.500 | 0.500 | 0.699 | 0.699 | 2.013 | 2.013 |
| | | 0.2 | 1.061 | 0.949 | 2.047 | 1.831 | 0.732 | 0.655 | 1.065 | 0.952 | 0.530 | 0.475 | 0.752 | 0.673 | 2.202 | 1.969 |
| | | 0.4 | 1.157 | 0.896 | 2.125 | 1.646 | 0.785 | 0.608 | 1.164 | 0.901 | 0.578 | 0.448 | 0.833 | 0.645 | 2.520 | 1.952 |
| | | 0.6 | 1.330 | 0.841 | 2.282 | 1.443 | 0.885 | 0.560 | 1.352 | 0.855 | 0.665 | 0.421 | 0.978 | 0.619 | 3.012 | 1.905 |
| | | 0.8 | 1.765 | 0.789 | 2.735 | 1.223 | 1.138 | 0.509 | 1.816 | 0.812 | 0.879 | 0.393 | 1.323 | 0.591 | 4.180 | 1.869 |
| | | 1.0 | 0.000 | 0.728 | 0.000 | 1.000 | 0.000 | 0.457 | 0.000 | 0.774 | 0.000 | 0.364 | 0.000 | 0.565 | 0.000 | 1.841 |
| 1.0 | 0.7 | 0.0 | 1.000 | 1.000 | 2.013 | 2.013 | 0.699 | 0.699 | 1.000 | 1.000 | 0.500 | 0.500 | 0.699 | 0.699 | 2.013 | 2.013 |
| | | 0.2 | 1.067 | 0.955 | 2.110 | 1.888 | 0.734 | 0.657 | 1.103 | 0.987 | 0.540 | 0.483 | 0.767 | 0.686 | 2.228 | 1.993 |
| | | 0.4 | 1.173 | 0.909 | 2.288 | 1.772 | 0.791 | 0.613 | 1.261 | 0.977 | 0.603 | 0.467 | 0.870 | 0.674 | 2.577 | 1.996 |
| | | 0.6 | 1.364 | 0.863 | 2.609 | 1.650 | 0.897 | 0.567 | 1.517 | 0.959 | 0.716 | 0.453 | 1.046 | 0.661 | 3.146 | 1.990 |
| | | 0.8 | 1.832 | 0.819 | 3.455 | 1.545 | 1.168 | 0.522 | 2.119 | 0.947 | 0.987 | 0.441 | 1.454 | 0.650 | 4.432 | 1.982 |
| | | 1.0 | 0.000 | 0.776 | 0.000 | 1.400 | 0.000 | 0.481 | 0.000 | 0.936 | 0.000 | 0.430 | 0.000 | 0.640 | 0.000 | 1.960 |

Table 1. Equivalent Length Factors for Various End Conditions

$$[K_1 = l_1(\text{eff}) / L_T; K_2 = l_2(\text{eff}) / L_T]$$

| END CONDITION (BOTTOM - TOP) | | | (1) PIN - PIN | | (2) FIX - FREE | | (3) FIX - PIN | | (4) FIX - SLIDER | | (5) FIX - FIX | | (6) PIN - FIX | | (7) PIN-SLIDER | |
|---------------------------------|-----|-----|---------------|-------|----------------|-------|---------------|-------|------------------|-------|---------------|-------|---------------|-------|----------------|-------|
| I1 | R2 | P2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 | K1 | K2 |
| -- | -- | -- | | | | | | | | | | | | | | |
| I2 | LT | PT | | | | | | | | | | | | | | |
| 1.0 | 0.9 | 0.0 | 1.000 | 1.000 | 2.013 | 2.013 | 0.699 | 0.699 | 1.000 | 1.000 | 0.500 | 0.500 | 0.699 | 0.699 | 2.083 | 2.083 |
| | | 0.2 | 1.097 | 0.981 | 2.198 | 1.966 | 0.759 | 0.679 | 1.117 | 0.999 | 0.558 | 0.499 | 0.781 | 0.699 | 2.236 | 2.000 |
| | | 0.4 | 1.242 | 0.962 | 2.508 | 1.943 | 0.851 | 0.659 | 1.292 | 1.001 | 0.642 | 0.498 | 0.901 | 0.698 | 2.592 | 2.008 |
| | | 0.6 | 1.493 | 0.944 | 3.089 | 1.954 | 1.012 | 0.640 | 1.579 | 0.998 | 0.785 | 0.496 | 1.104 | 0.698 | 3.294 | 2.083 |
| | | 0.8 | 2.086 | 0.933 | 4.262 | 1.906 | 1.391 | 0.622 | 2.231 | 0.998 | 1.108 | 0.495 | 1.560 | 0.698 | 4.659 | 2.083 |
| | | 1.0 | 0.000 | 0.909 | 0.000 | 1.800 | 0.000 | 0.605 | 0.000 | 0.997 | 0.000 | 0.495 | 0.000 | 0.695 | 0.000 | 1.998 |



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